Clarkson University defy convention

Coastal Dynamics Modeling

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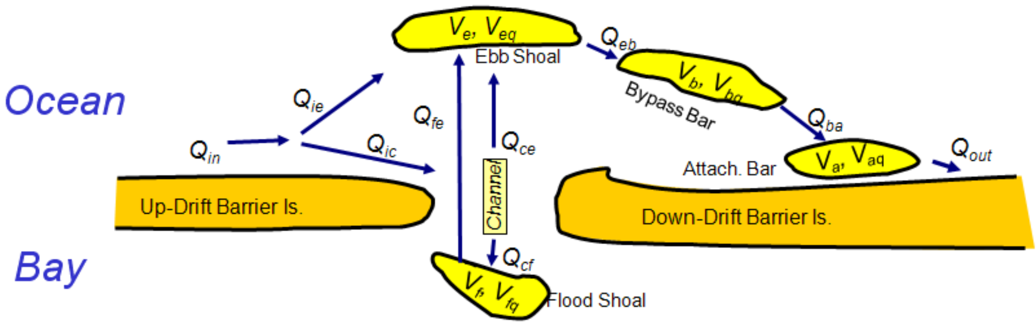
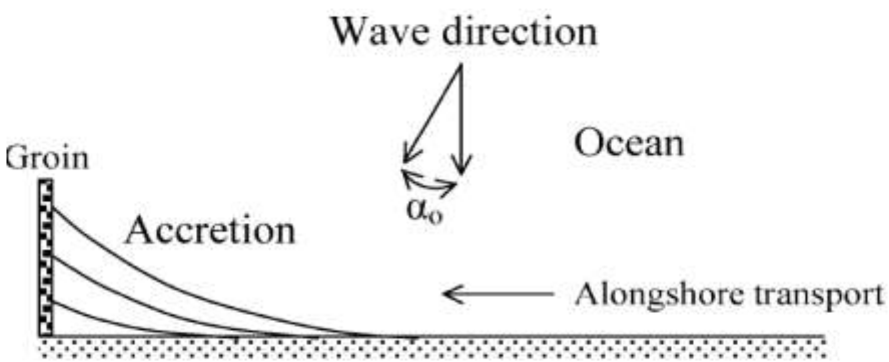
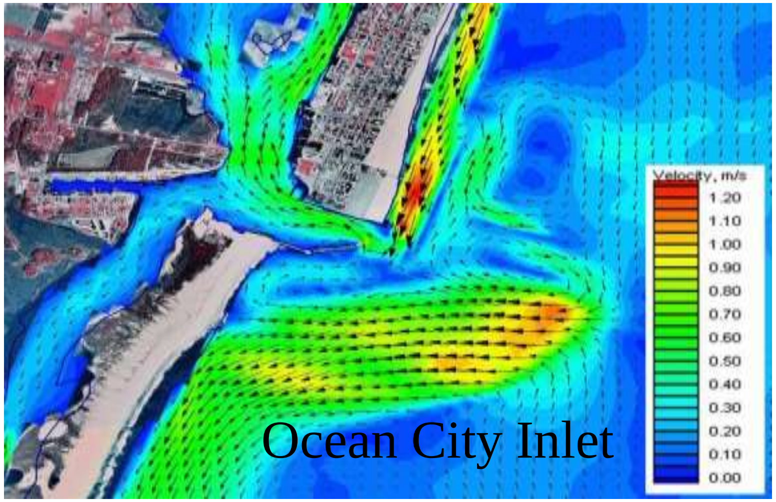
Entrance of Humboldt Bay, CA

Coastal Morphodynamic Modeling

- Conceptual models
- Shoreline evolution models
- Beach profile evolution models
- Coastal area or 2DH models
- Quasi-3D models
- 3D models







CMS (Coastal Modeling System)

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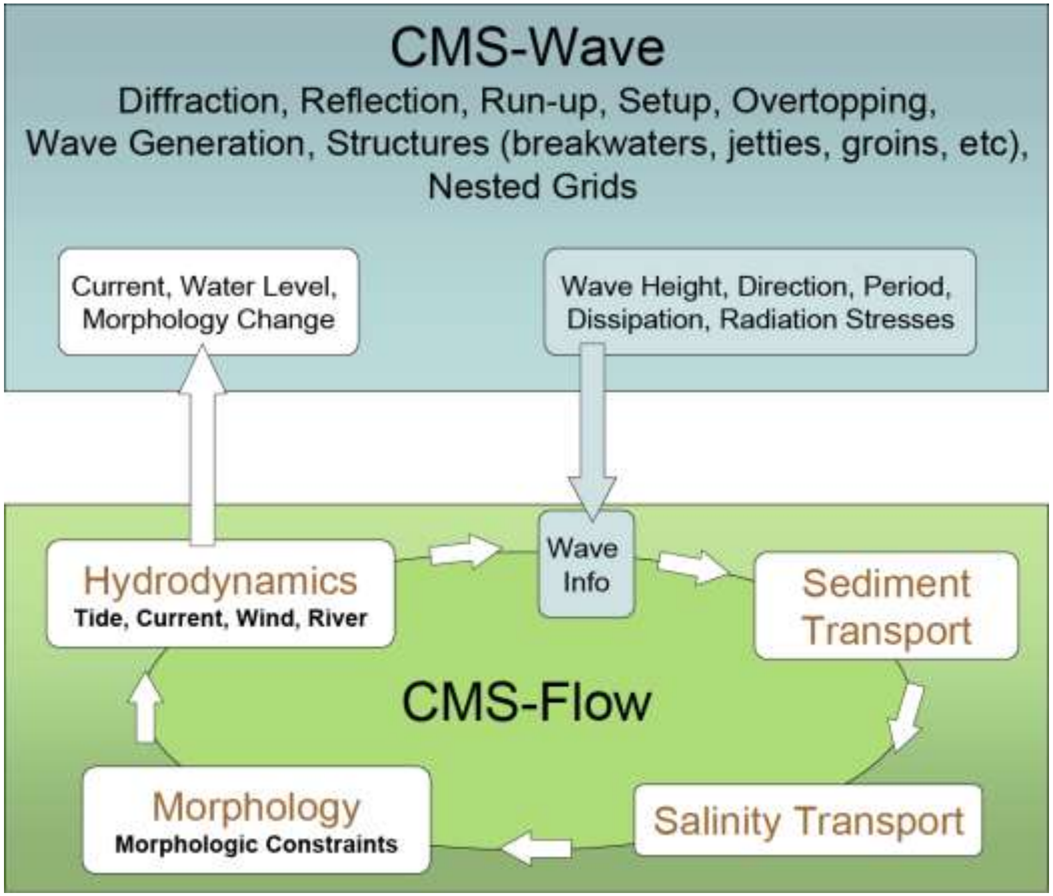
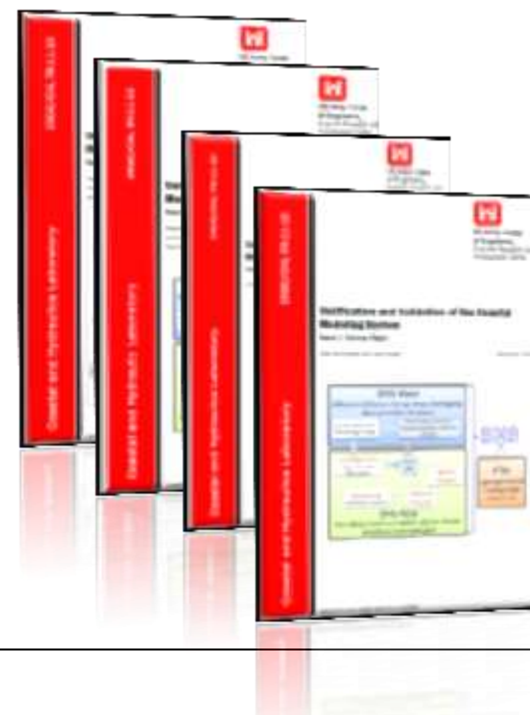
Coastal Modeling System (CMS)



- Integrated system
- Inline code
- Parallelization on PC's
- SMS Interface
- Verification & Validation reports available: http://cirp.usace.army.mil



Decomposition of Velocity

- Velocity split

Current, Wave, Turbulent

$$\hat{u}_i = u_i + \tilde{u}_i + u_i' \qquad \langle u_i' \rangle = 0$$

- Total flux

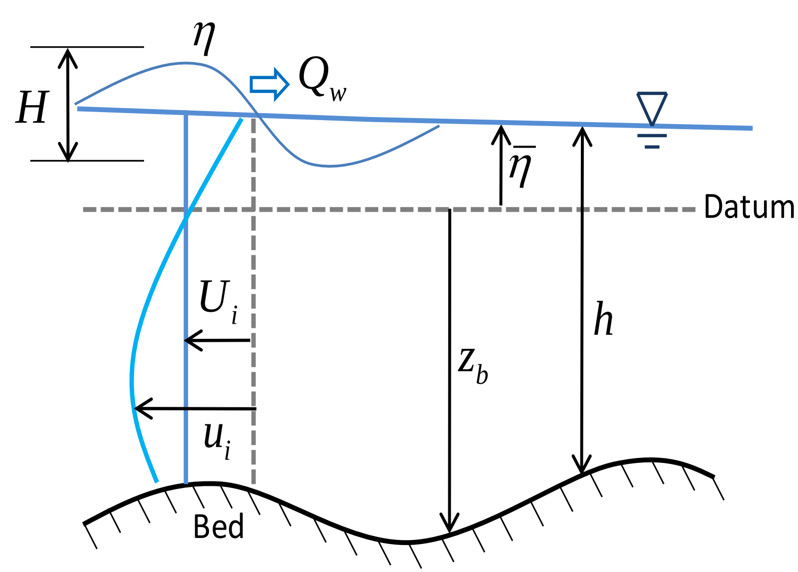
$$hV_i = \overline{\int_{z_b}^{\eta} \hat{u}_i dz} \qquad V_i = U_i + U_{wi}$$

- Wave flux

$$Q_{wi} = hU_{wi} = \overline{\int_{\eta_t}^{\eta} \hat{u}_i dz}$$

- Current velocity

$$hU_i = \overline{\int_{z_b}^{\eta} u_i dz} = \int_{z_b}^{\bar{\eta}} u_i dz$$



Hydrodynamics

- Continuity

$$\frac{\partial h}{\partial t} + \frac{\partial (hV_j)}{\partial x_j} = 0$$

$$V_i = U_i + U_{wi}$$

Contribution

- Momentum

Temporal, Advection, Coriolis-Stokes, Wave level Gradient, Atmospheric Pressure, Wind stress

$$\frac{\partial (hV_i)}{\partial t} + \frac{\partial (hV_iV_j)}{\partial x_j} - \varepsilon_{ij} f_c hV_j = -gh\frac{\partial \bar{\eta}}{\partial x_i} - \frac{h}{\rho}\frac{\partial p_a}{\partial x_i}$$

$$+ \frac{\partial}{\partial x_j}\left(\nu_t h \frac{\partial V_i}{\partial x_j}\right) - \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(S_{ij} + R_{ij} - \rho h U_{wi} U_{wj}\right) + \frac{\tau_{si}}{\rho} - m_b \frac{\tau_{bi}}{\rho}$$

Mixing, Radiation stress, Roller stress, Excess Wave momentum, Bottom friction

Turbulent Eddy Viscosity

$$\nu_t = \nu_0 + \nu_c + \nu_w$$

- Base or background value

$$\nu_0 = 1\times 10^{-6} - 0.1$$

- Wave-related component
  - Modified Kraus and Larson (1991)

$$\nu_w = c_{wf} u_{ws} H_s + c_{br} h \left(\frac{D_{br}}{\rho} \right)^{1/3}$$

- Current-related component
  - Subgrid model

$$\nu_c = c_v u_{*c} h + (c_h \Delta)^2 \left| \bar{S} \right| \qquad \left| \bar{S} \right| = \sqrt{2 e_{ij} e_{ij}} \qquad e_{ij} = \frac{1}{2} \left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right)$$

  - Mixing length model

$$\nu_c = \sqrt{\left(c_v u_{*c} h \right)^2 + \left(l_h^2 \left| \bar{S} \right| \right)^2} \qquad l_h = \kappa \min\left(c_m h, y' \right)$$

Mean Bottom Shear Stress

- Wave-current bottom friction

$$\tau_b = \rho c_b U_i \sqrt{U^2 + c_w u_{ws}^2} \qquad c_w = 0.65$$

- Bottom wave orbital velocity

$$u_{ws} = \frac{\pi H_s}{T_p \sinh(kh)}$$

- Bottom friction coefficient

$$c_b = g n^2 h^{-1/3} \qquad c_b = \left[\frac{\kappa}{\ln(h / z_0) - 1} \right]^2$$

- For currents only reduces to

$$\tau_{bc} = \rho c_b U U_i$$

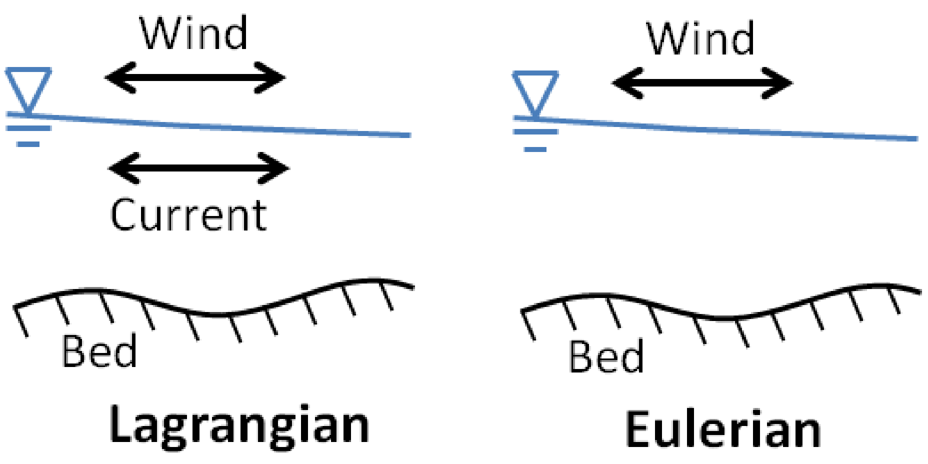
Surface Shear Stress

- Wind reference frame

$$\tau_{si} = \rho_a C_D W W_i$$

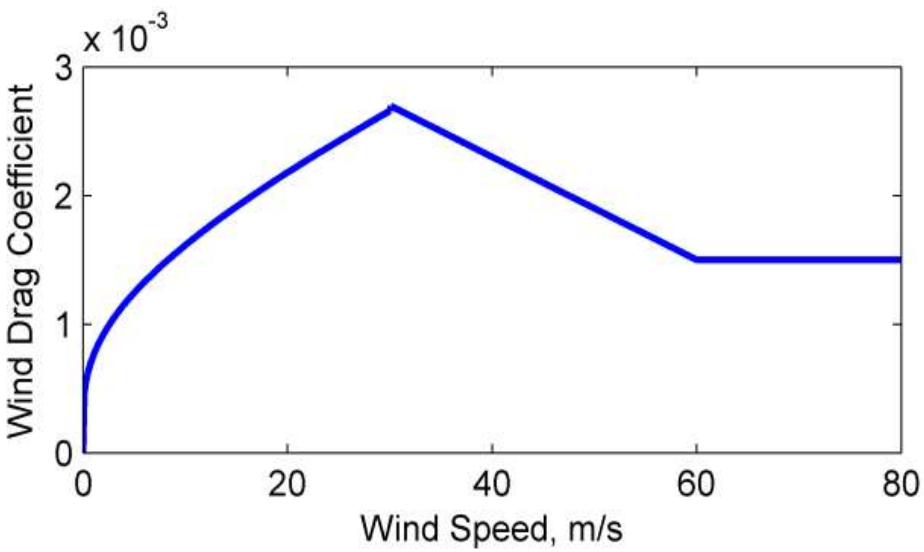
$$W_i = W_i^E - \gamma_W U_i$$

$$\gamma_W = \begin{cases} 0 \text{ for Eulerian reference frame} \\ 1 \text{ for Lagrangian reference frame} \end{cases}$$



- Modified Hsu (1988) drag coefficient

$$C_D = \begin{cases} \left(\dfrac{\kappa}{14.56 - 2\ln W}\right)^2 & \text{for } W \le 30\,\text{m/s} \\ 10^{-3}\max(3.86 - 0.04W, 1.5) & \text{for } W > 30\,\text{m/s} \end{cases}$$



Boundary Conditions

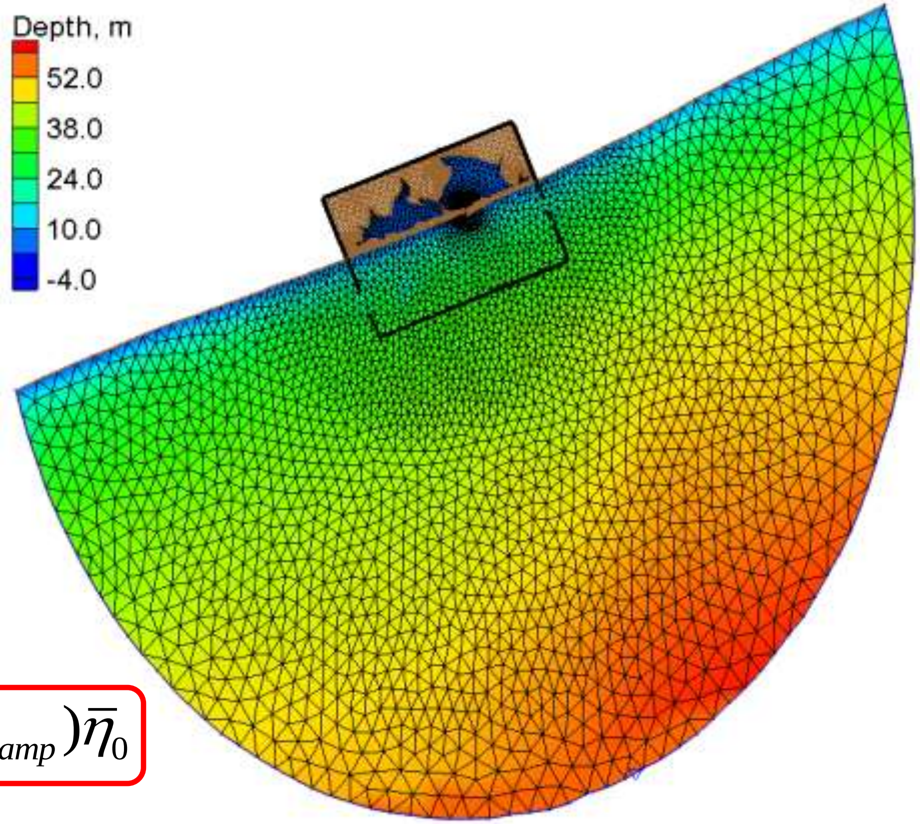
- Wall Boundary

$$F_f = 0 \qquad \tau_{wall} = \rho c_{wall} U_{\parallel}^2 \qquad c_{wall} = \left[\frac{\kappa}{\ln(y_P / y_0)} \right]^2 \qquad \vec{U}_{\parallel} = \vec{U} - (\vec{U} \cdot \vec{n})\vec{n}$$

- Flux Boundary

$$Q_{tot} = \sum_B h_B (\vec{U}_B \cdot \hat{n}) \Delta l_B$$

$$\vec{U}_B = \frac{f_{Ramp} Q}{\left| \sum_B (\hat{e} \cdot \hat{n})_B \frac{h_B^{r+1}}{n_B} \Delta l_B \right|} \frac{h_B^r}{n_B} \hat{e}_B$$



- Water level Boundary

$$\bar{\eta}_B = f_{Ramp} (\bar{\eta}_E + \Delta \bar{\eta} + \bar{\eta}_C + \bar{\eta}_G) + (1 - f_{Ramp}) \bar{\eta}_0$$

Cross-Shore Boundary Conditions

The alongshore (y) component of velocity is calculated by solving the following equation:

$$0 = \frac{\partial}{\partial x}\left(\nu_t h \frac{\partial V}{\partial x} \right) + \tau_{sy} + \tau_{wy} - c_b U_{cw} V$$

The cross-shore (x) component of velocity is copied from internal node;

The alongshore gradient of water level is zero, or the water level is determined by

$$0 = -\rho g \frac{\partial \eta}{\partial x} + \tau_{sx} + \tau_{wx}$$

CMS-Wave

- Spectral wave-action balance equation

$$\frac{\partial\ c_x N}{\partial x}+\frac{\partial\ c_y N}{\partial y}+\frac{\partial\ c_\theta N}{\partial \theta}=\frac{K}{2\sigma}\left[\frac{\partial}{\partial y}\left(cc_g\cos^2\theta\frac{\partial N}{\partial y}\right)-\frac{cc_g}{2}\cos^2\theta\frac{\partial^2 N}{\partial y^2}\right]-\varepsilon_b N-S$$

- Characteristic velocities

$$N=\frac{E(f,\theta)}{\sigma}$$

$$c_x=c_g\cos\theta+U_x \qquad c_y=c_g\sin\theta+U_y$$

$$c_\theta=\frac{\sigma}{\sinh 2kh}\left(\sin\theta\frac{\partial h}{\partial x}-\cos\theta\frac{\partial h}{\partial y}\right)+\cos\theta\left(\sin\theta\frac{\partial U_x}{\partial x}-\cos\theta\frac{\partial U_x}{\partial y}\right)+\sin\theta\left(\sin\theta\frac{\partial U_y}{\partial x}-\cos\theta\frac{\partial U_y}{\partial y}\right)$$

- Dispersion relation

$$\sigma^2=gk\tanh(kh) \qquad \sigma=\omega-\vec{k}\cdot\vec{U}$$

- Radiation stresses

$$S_{ij}=\iint E_w(f,\theta)\left[n_g w_i w_j+\delta_{ij}\left(n_g-\frac{1}{2}\right)\right]df\,d\theta$$

Surface Roller

- As wave transitions from nonbreaking to breaking, part of the energy goes into the aerated region known as surface roller as momentum and later transferred to the flow below
- Roller energy balance

$$\frac{\partial(2E_{sr}c_j)}{\partial x_j} = -D_r + f_e D_{br}$$

$E_{sr} \rightarrow$ Rolle energy density

$D_{br} \rightarrow$ Wave breaking dissipation

$D_{sr} \rightarrow$ Surface roller dissipation

$f_e \rightarrow$ Efficiency factor

- Assumptions
  - Roller direction in same direction as waves
  - Nonspectral
- Roller dissipation

$$D_{sr} = \frac{g 2E_{sr}\beta_D}{c}$$

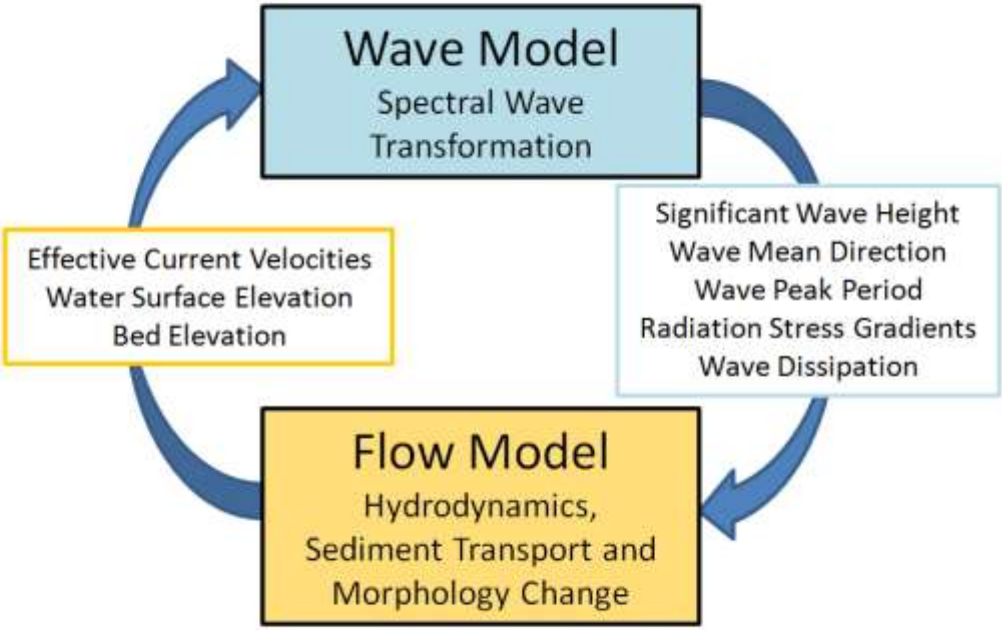
$\beta_D \rightarrow$ Roller dissipation coefficient

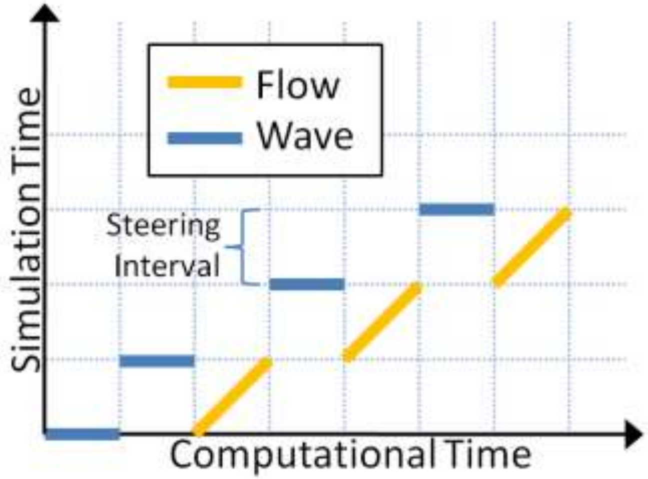
- Roller stress

$$R_{ij} = 2E_{sr} w_i w_j$$

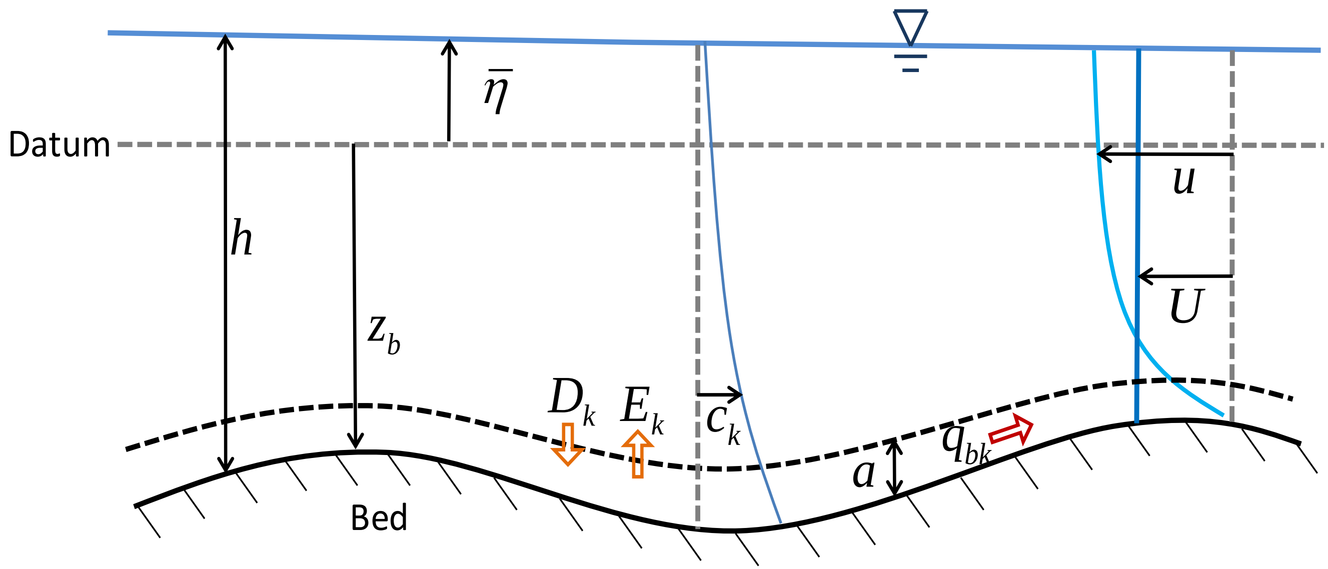
Coupling between Flow and Waves

- Steering process
  - Roller included in wave model
  - Sediment transport and morphology change included in flow model
- Flow and Wave models may have the same or different grids





Sediment Transport Model



$$r_{sk} = \frac{C_k}{C_{tk}} \approx \frac{C_{k*}}{C_{tk*}}$$

$$C_{tk} = C_k + \frac{q_{bk}}{Uh}$$

- Total load transport equation

$$\frac{\partial}{\partial t}\left(\frac{hC_{tk}}{\beta_{tk}}\right) + \frac{\partial\left(hU_j C_{tk}\right)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\nu_s h \frac{\partial\left(r_{sk} C_{tk}\right)}{\partial x_j}\right] + \alpha_t \omega_{sk}\left(C_{t*k} - C_{tk}\right)$$

- Bed change equation

$$\frac{\partial z_b}{\partial t} = \sum_k \left(\frac{\partial z_b}{\partial t}\right)_k \qquad (1 - p'_m)\left(\frac{\partial z_b}{\partial t}\right)_k = \alpha_t \omega_{sk}\left(C_{tk} - C_{t*k}\right) + \frac{\partial}{\partial x_j}\left(D_s q_{bk} \frac{\partial z_b}{\partial x_j}\right)$$

Total-load Correction Factor

$$\beta_{tk} = \frac{1}{r_{sk} \big/ \beta_{sk} + (1 - r_{sk})U \big/ u_{bk}}$$

- Suspended-load correction factor $\beta_{sk} = \dfrac{\int_{z_b+a}^{\bar{\eta}} u c_k dz}{U \int_{z_b+a}^{\bar{\eta}} c_k dz}$
- Assuming logarithmic velocity and exponential concentration profiles

$$\beta_{sk} = \frac{\mathrm{E}_1(\phi_k A) - \mathrm{E}_1(\phi_k) + \ln(A/Z)\mathrm{e}^{-\phi_k A} - \ln(1/Z)\mathrm{e}^{-\phi_k}}{\mathrm{e}^{-\phi_k A}\left[\ln(1/Z) - 1\right]\left[1 - \mathrm{e}^{-\phi_k(1-A)}\right]}$$

$$\phi_k = \omega_{sk} h / \varepsilon \qquad A = a/h \qquad Z = z_a / h \qquad \mathrm{E}_1(x) = \int_x^{\infty} \frac{e^{-t}}{t} dt$$

Sediment Transport Formulas

- Lund-CIRP (Wu and Lin 2011)

$$\frac{q_{bk*}}{\sqrt{(s-1)gd_k^3}} = f_b \xi_k^{-1} p_{1k} \rho_s 12 \sqrt{\Theta_c} \Theta_{cw,m} \exp\left(-4.5 \frac{\Theta_{crk}}{\Theta_{cw}}\right) \qquad \frac{q_{sk*}}{\sqrt{(s-1)gd_k^3}} = f_s \xi_k^{-1} p_{1k} \rho_s c_{Rk} U \frac{\varepsilon_k}{\omega_{sk}} \left[1 - \exp\left(-\frac{\omega_{sk} h}{\varepsilon}\right)\right]$$

- Soulsby-van Rijn

$$q_{bk*} = f_b \rho_s p_{1k} 0.015 U h \left(\frac{U_e - \xi_k^{1/2} U_{crk}}{\sqrt{(s-1)gd_k}}\right)^{1.5} \left(\frac{d_k}{h}\right)^{1.2} \qquad q_{sk*} = f_s \rho_s p_{1k} 0.012 U h \left(\frac{U_e - \xi_k^{1/2} U_{crk}}{\sqrt{(s-1)gd_k}}\right)^{2.4} \left(\frac{d_k}{h}\right) d_{*k}^{-0.6}$$

- van Rijn

$$q_{bk*} = f_b \rho_s p_{1k} 0.005 U h \left(\frac{U_e - \xi_k^{1/2} U_{crk}}{\sqrt{s-1)gd_k}}\right)^{2.4} \left(\frac{d_k}{h}\right)^{1.2} \qquad q_{sk*} = f_s \rho_s p_{1k} 0.012 U h \left(\frac{U_e - \xi_k^{1/2} U_{crk}}{\sqrt{s-1)gd_k}}\right)^{2.4} \left(\frac{d_k}{h}\right) d_{*k}^{-0.6}$$

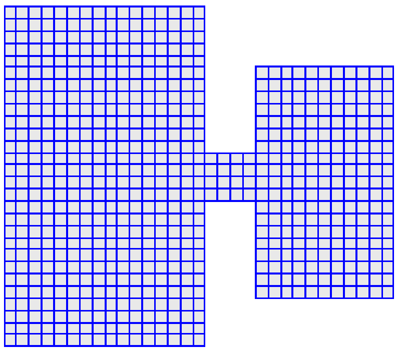
- Watanabe

$$q_{tk*} = \left[f_s r_{sk} + f_b (1 - r_{sk}) \right] \rho_s p_{1k} A_{Wat} U \left(\frac{\tau_{b\max} - \xi_k \tau_{crk}}{\rho g}\right)$$

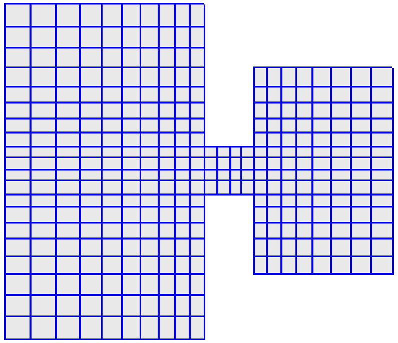
- Wu and Lin (2014)

Grids Used

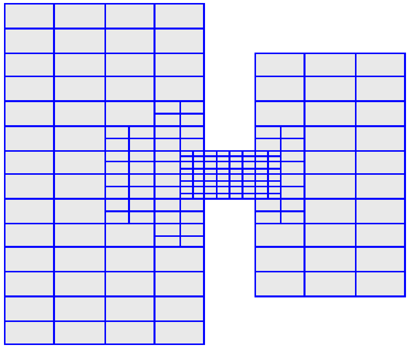
Regular
Cartesian



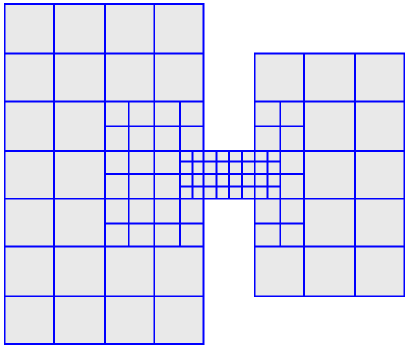
Nonuniform
Cartesian



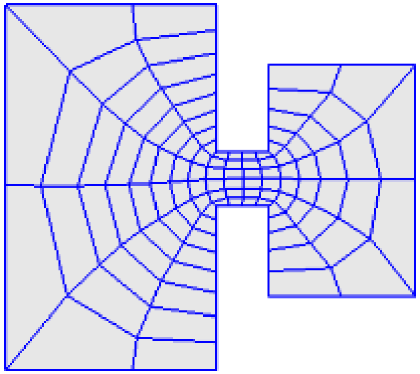
Telescoping
Cartesian



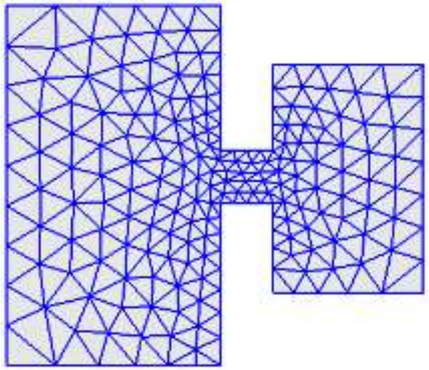
Stretched
Telescoping
Cartesian



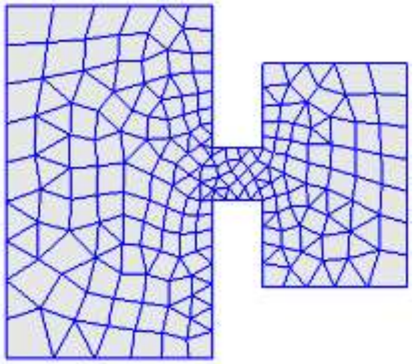
Quadrilateral
(Un)Structured



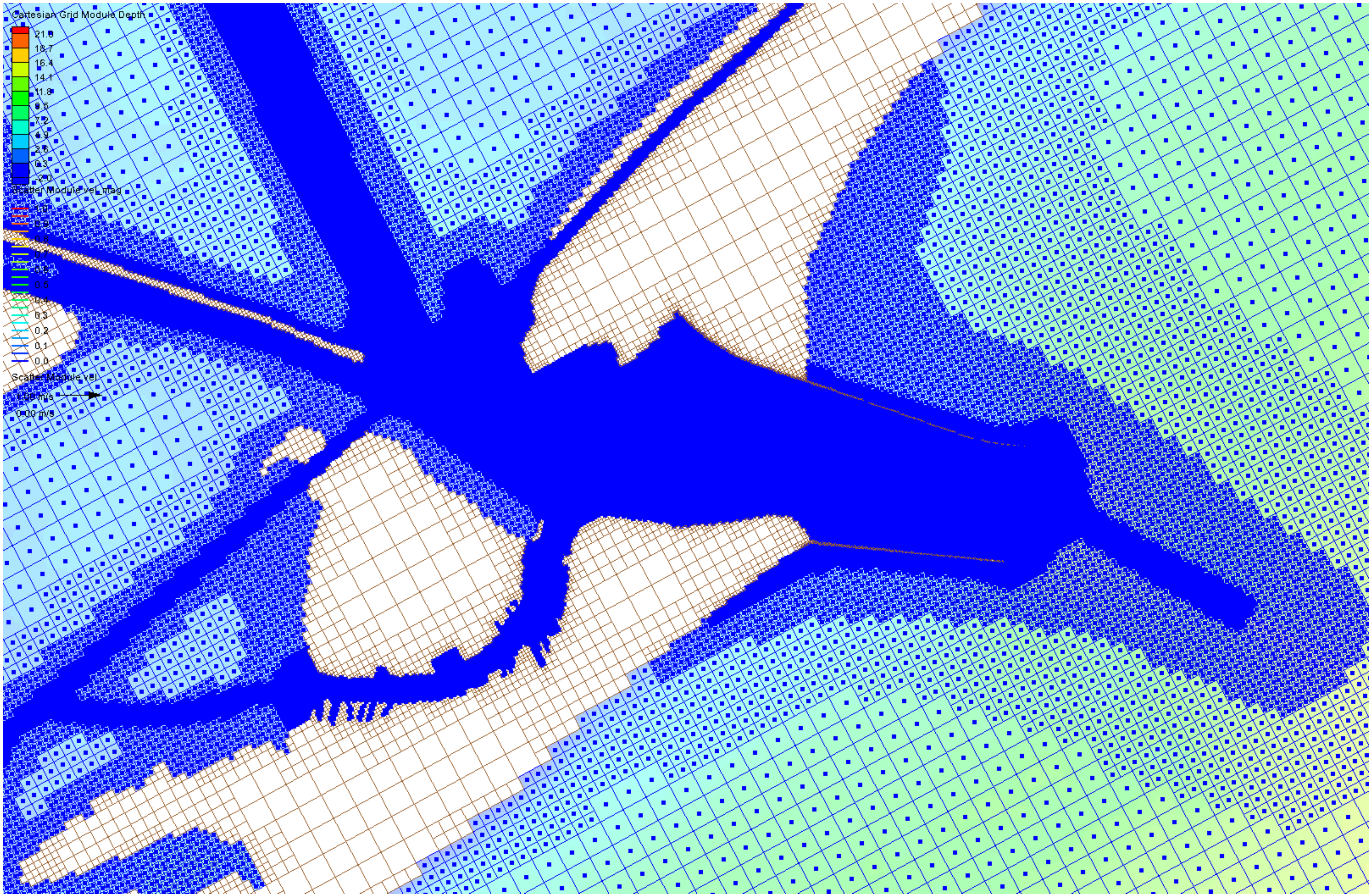
Triangular
Unstructured



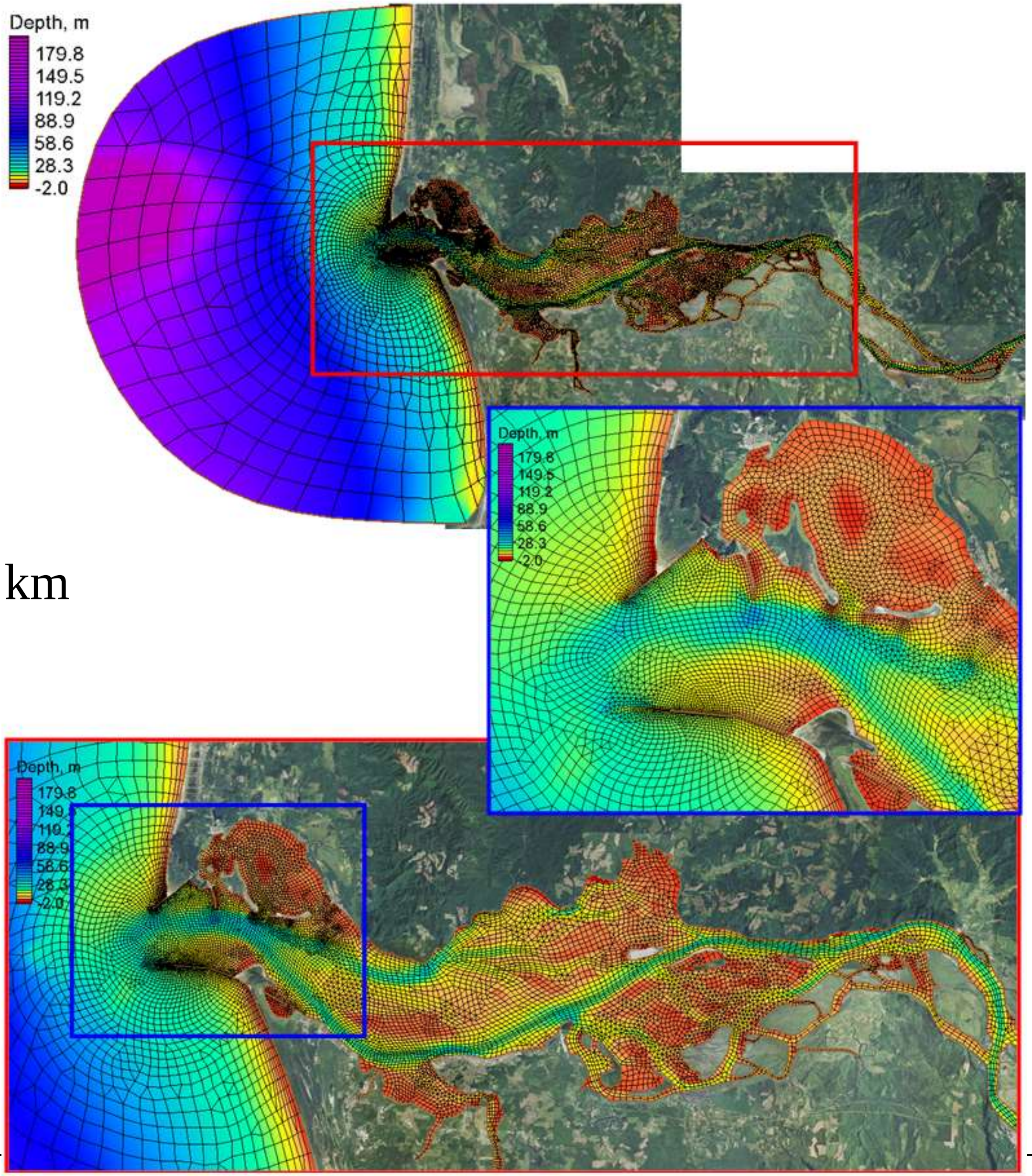
Hybrid
Unstructured



Galveston Entrance Channel, TX



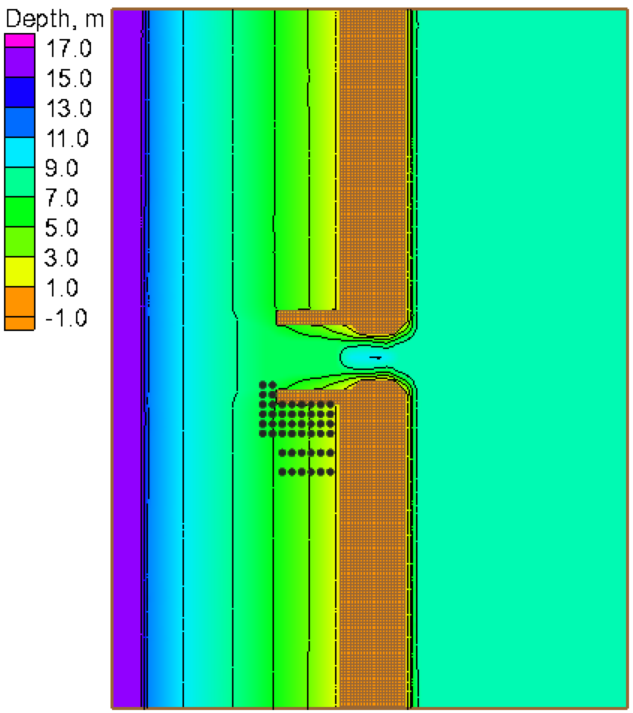
Columbia River, USA

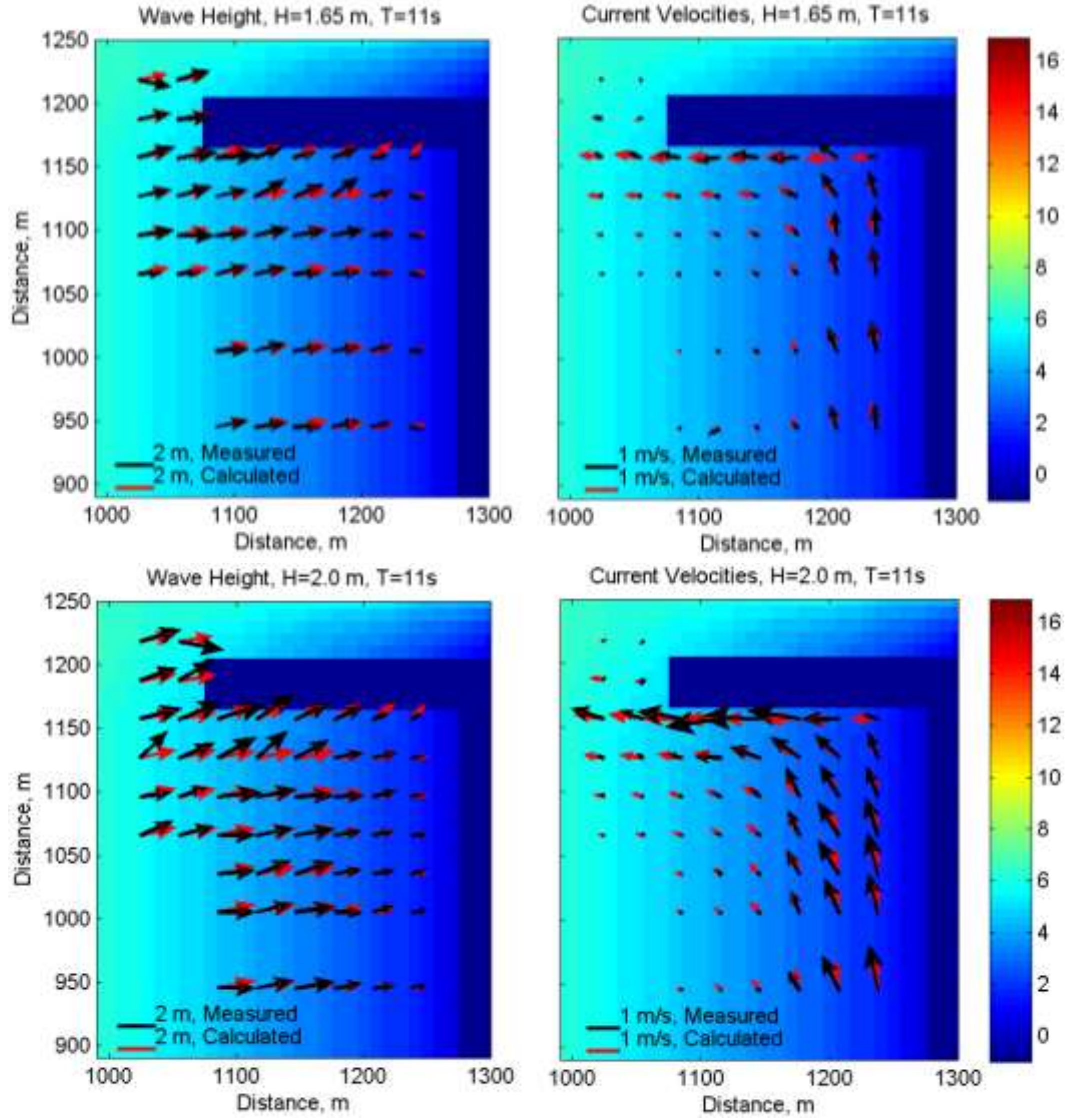


Hybrid mesh

- ~16k cells
- 20 m to 3.5 km resolution

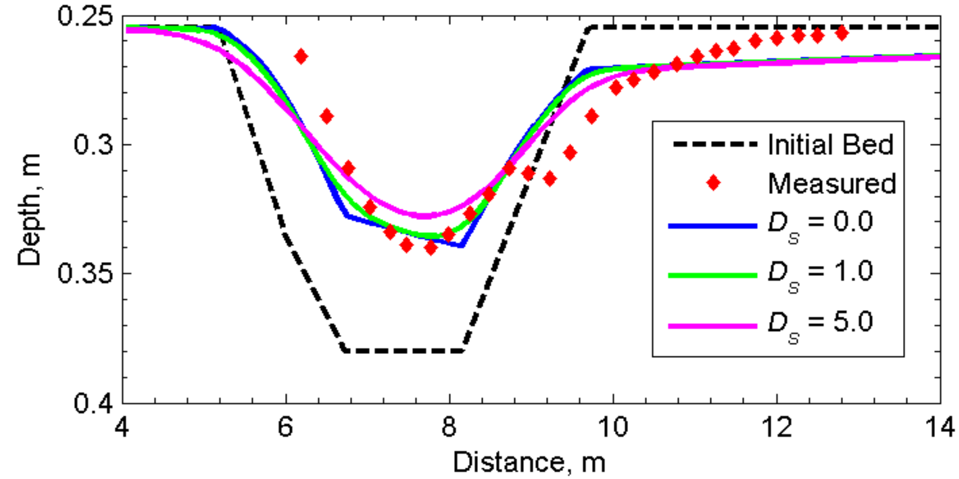
Laboratory Study of an Idealized Inlet

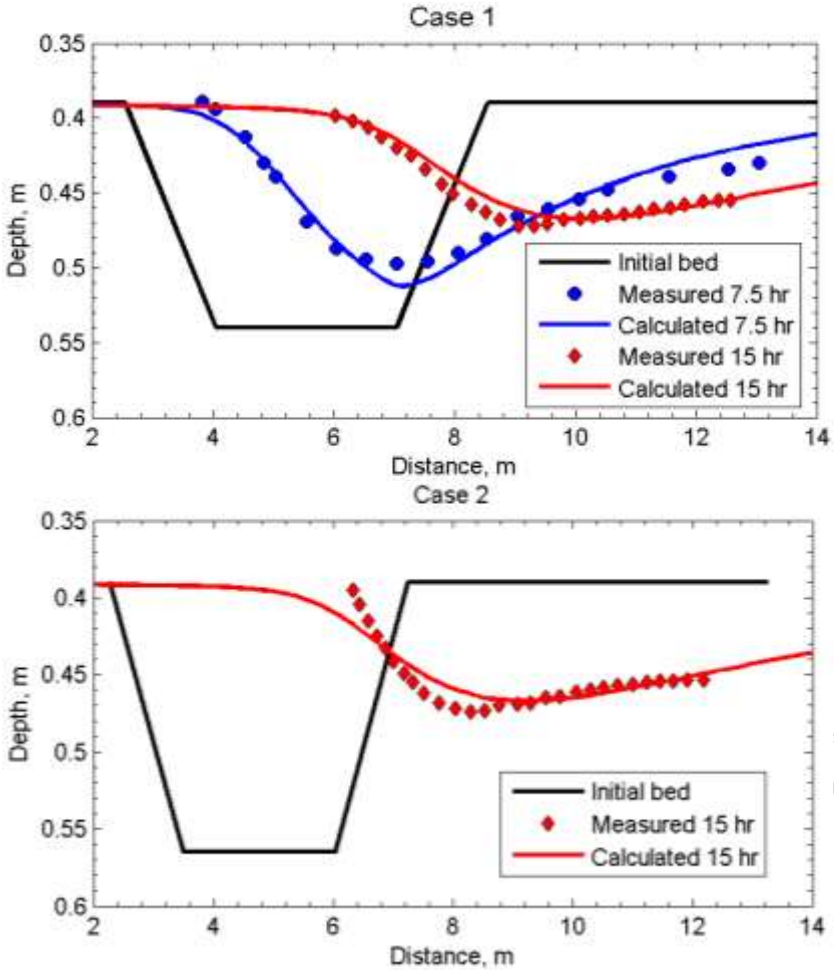


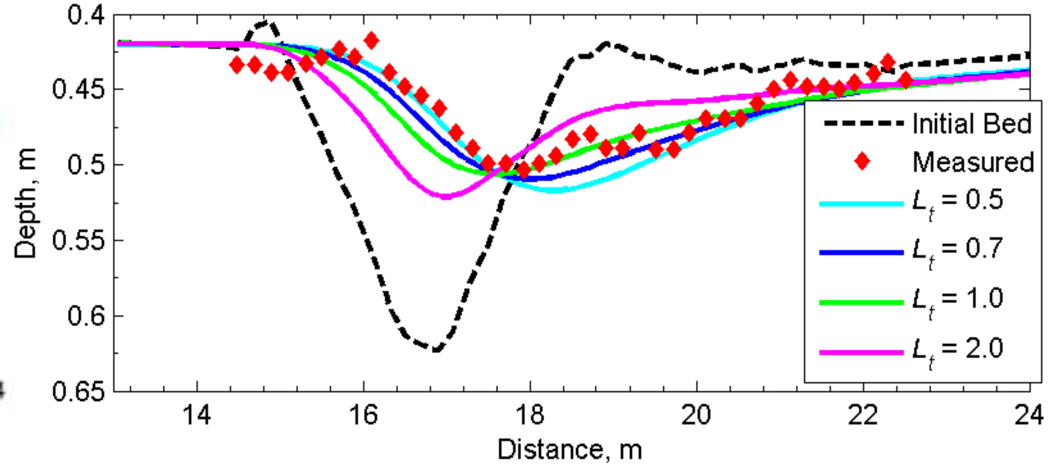


Channel Infilling

Van Rijn (1986)





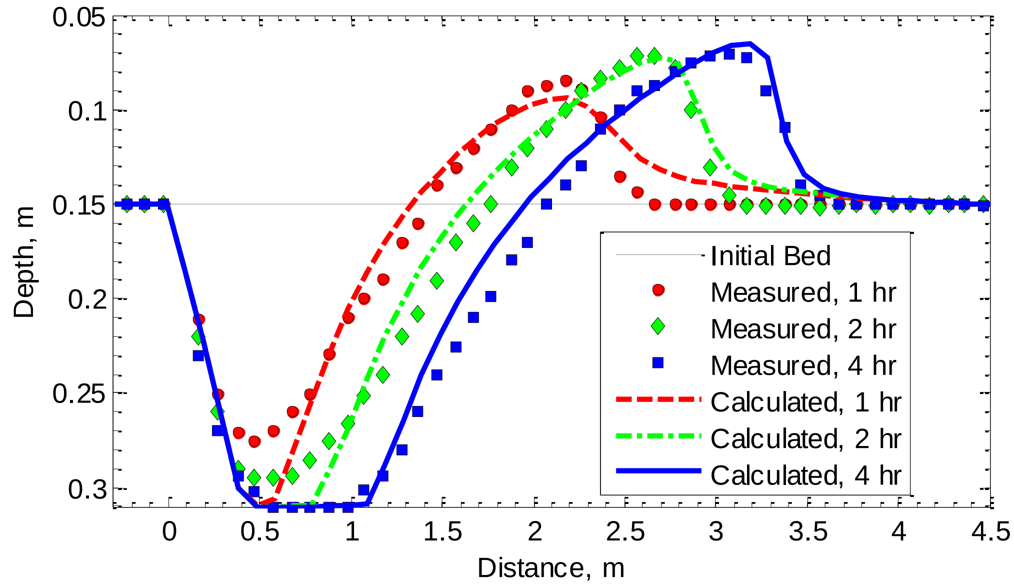


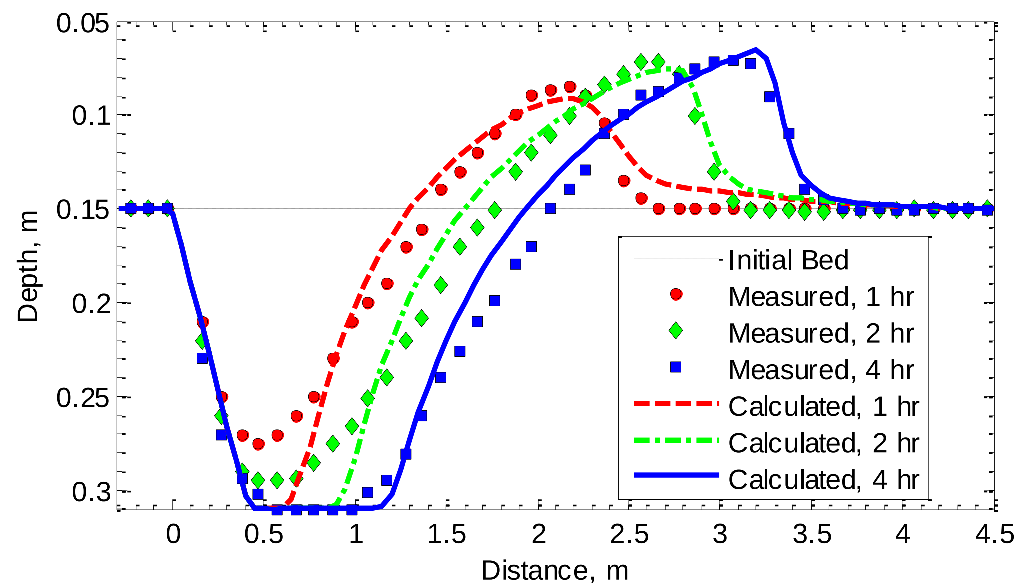
DHL (1980)

Van Rijn and Havinga (1995)

Clear Water Jet over a Hard Bottom

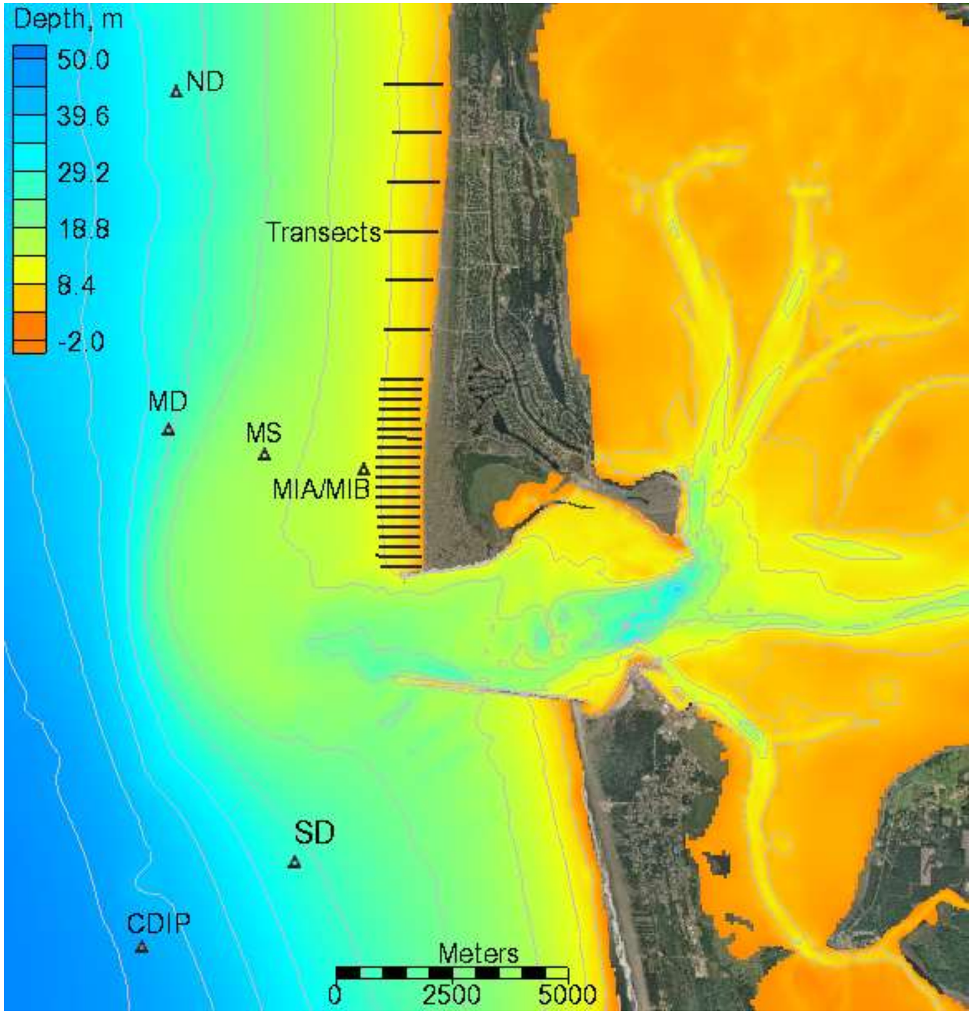
- Initial depth: 0.15 m
- D50: 0.6 mm
- Hard bottom: 0.31 m
- Inflow velocity: 0.6 m/s
- Time step: 30 sec
- Simulation duration: 4.25 hrs
- Manning's coefficient: 0.03 s/m^3
- Transport formula: Soulsby-van Rijn





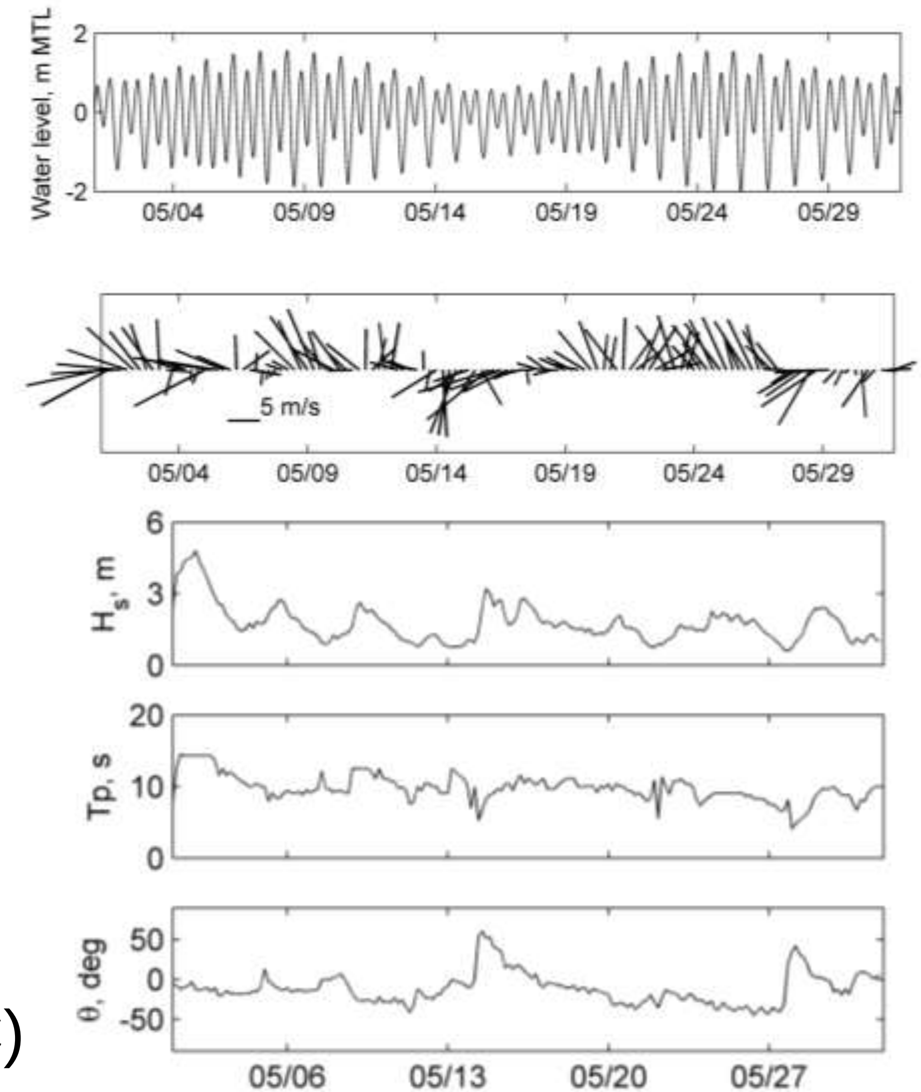
Grays Harbor, WA 2001 Field Study

- 6 Tripods deployed from May 5 to July 2001
- Weekly topo and monthly bathy surveys along 50-200 m spaced transects
- Grab sediment samples taken at tripod locations



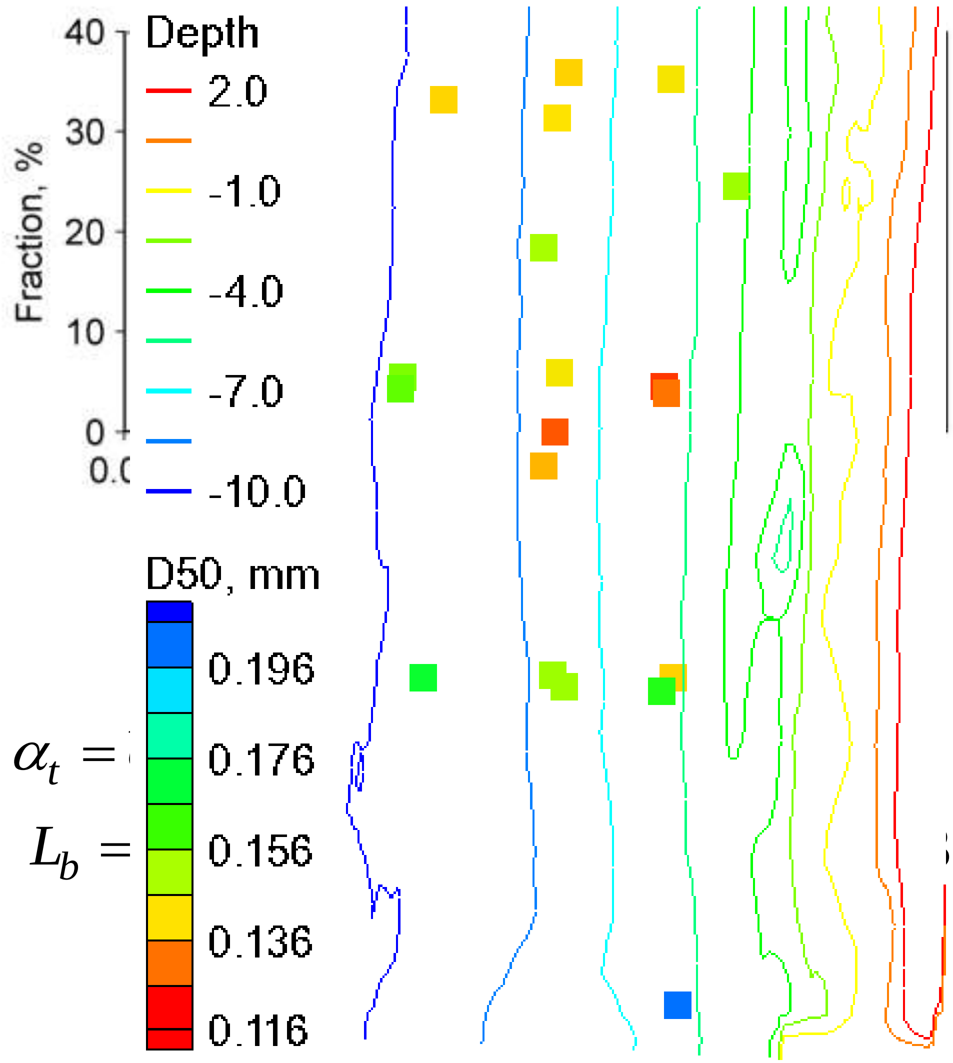
Model Setup

- Forcing
  - Tide from Westport Harbor (corrected phase)
  - Winds from NCDC Blended Sea Winds
  - Waves from CDIP buoy (42 m depth)
  - River flows from USGS
- Hydro and sediment transport
  - 10 min time step
  - Ramp of 5 days
- Waves
  - 2 hr steering interval
- 30 day simulation (~10 hrs PC)

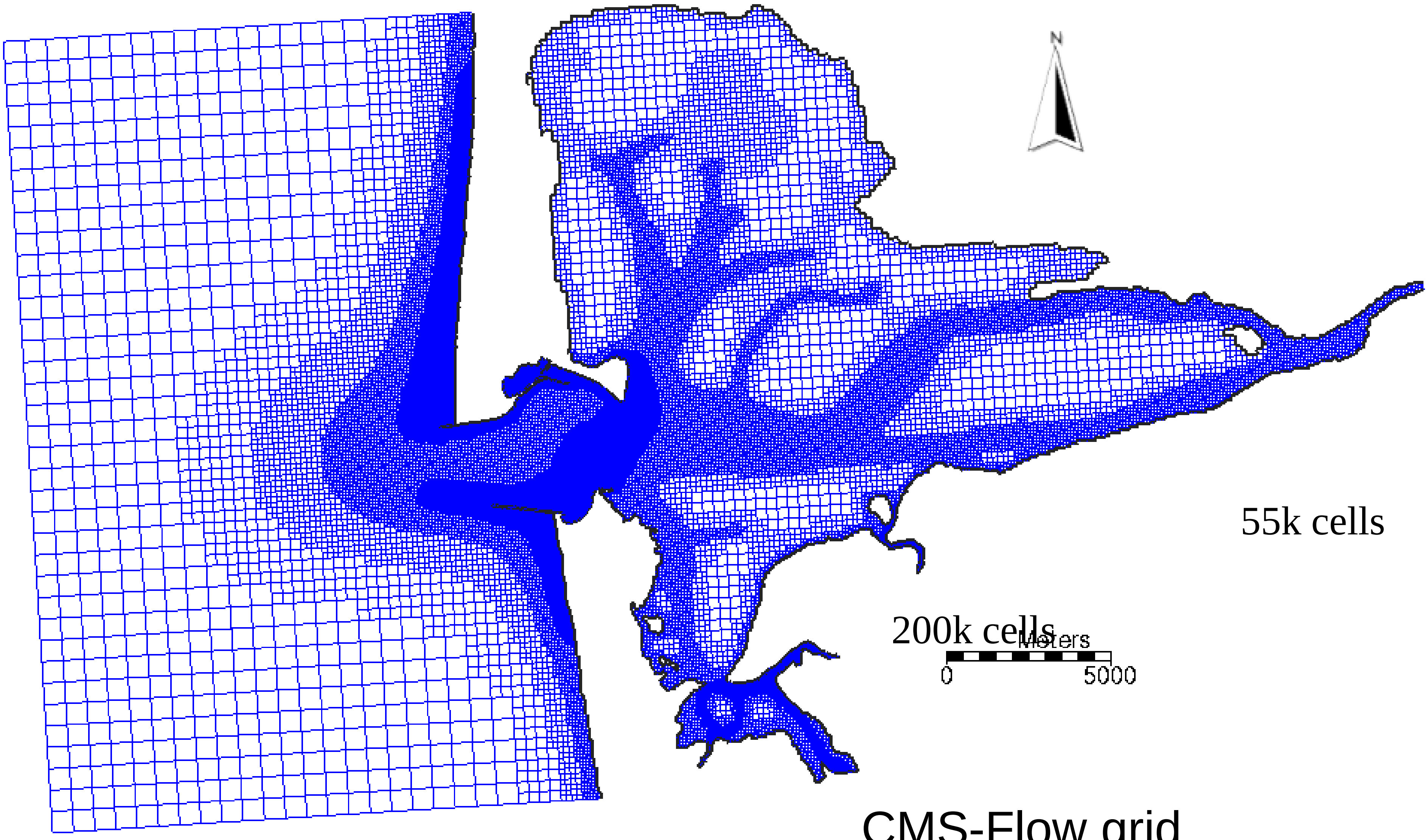


Sediment Transport Setup

- Grain size distributions?
  - Expect coarser on bar and beach face and finer in trough
  - No before and after
- Spatially constant initial grain size distribution
- Lund-CIRP transport formula
- Bed porosity = 0.3
- Adaptation coefficient
- 10 bed layers
- Initial thickness of second layer and below = 0.5 m

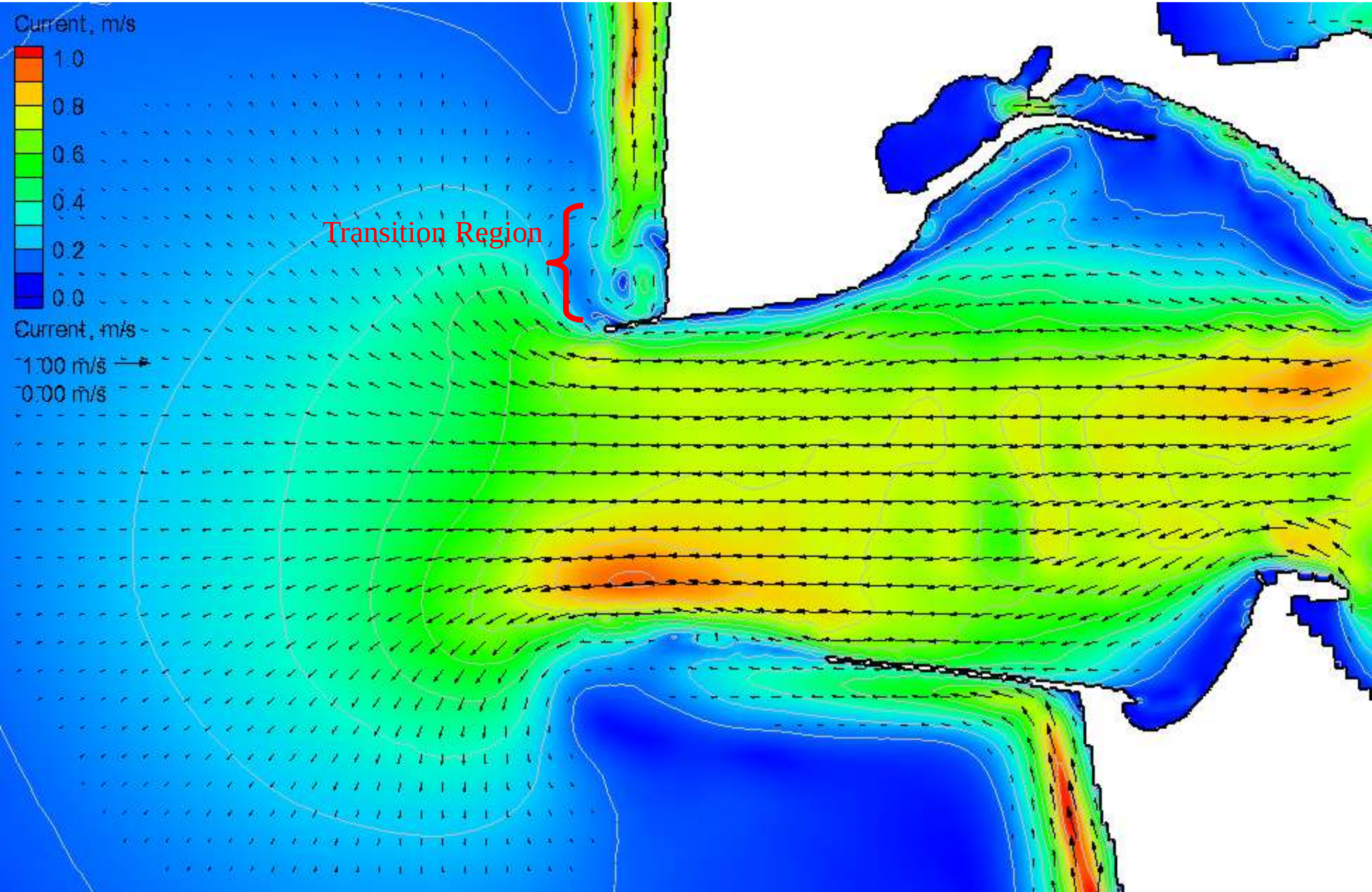


Computational Grid



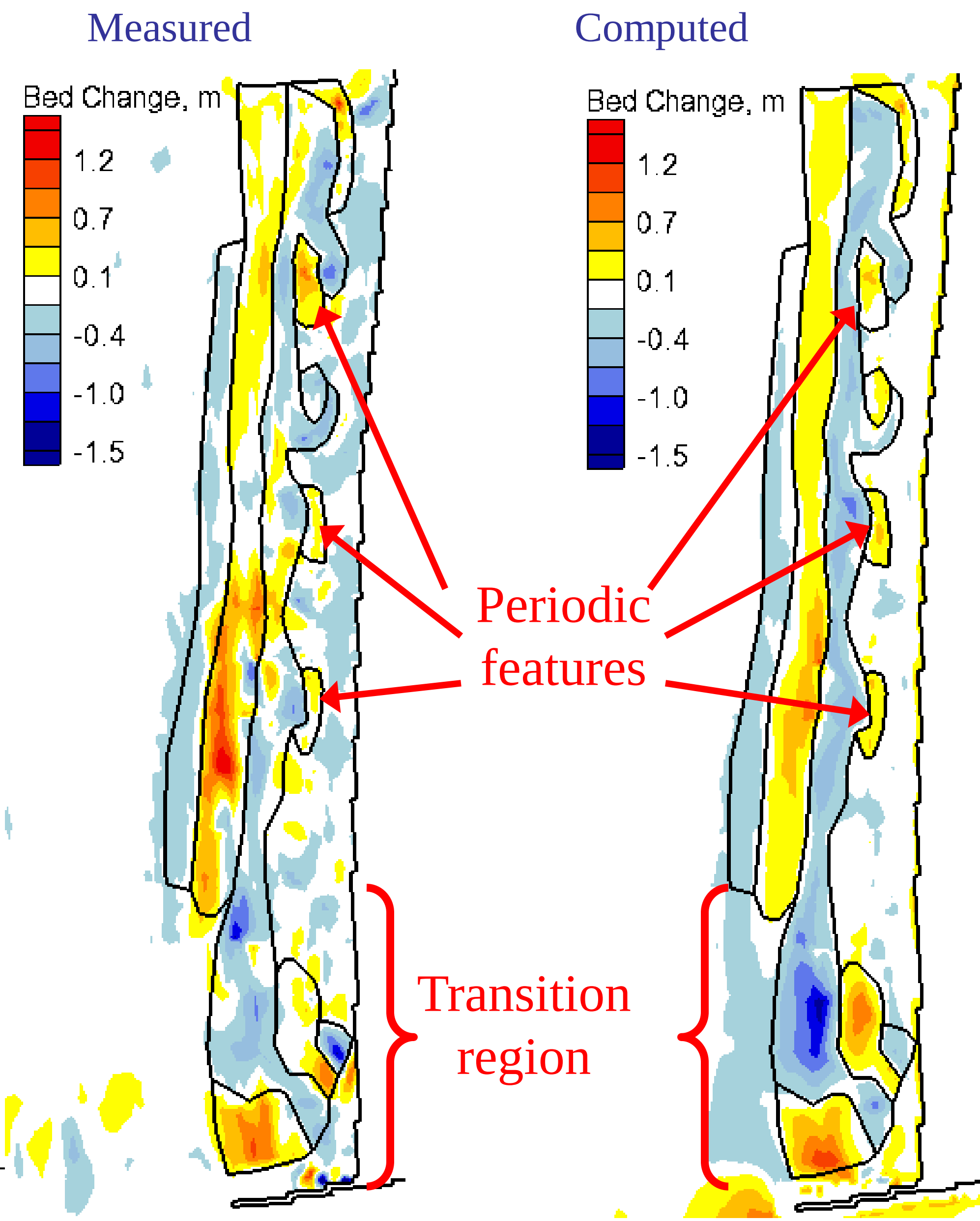
CMS-Flow grid

Current Velocities (May 14, 2001)

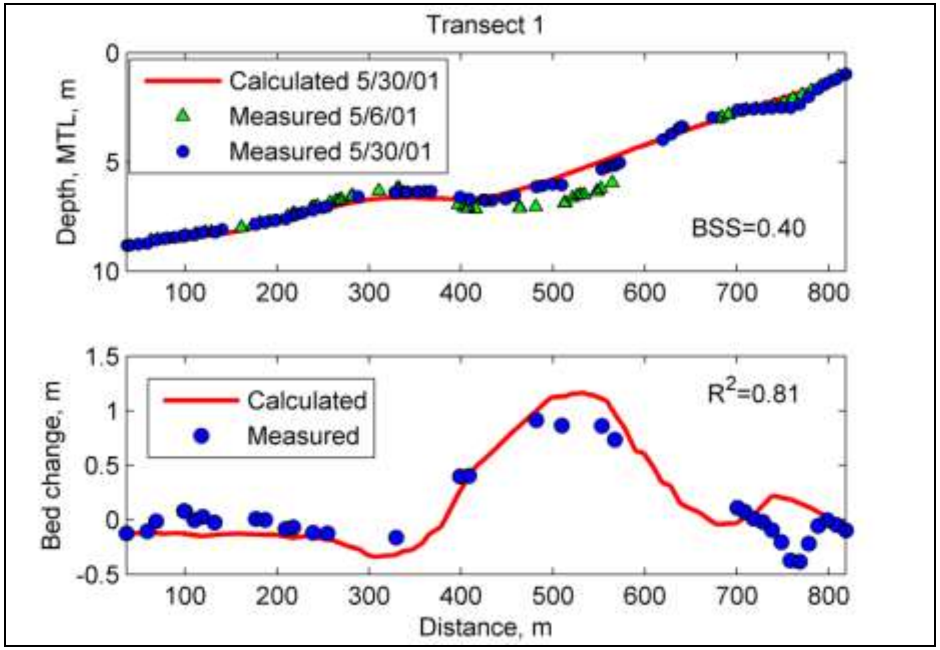


Results - Bed Change

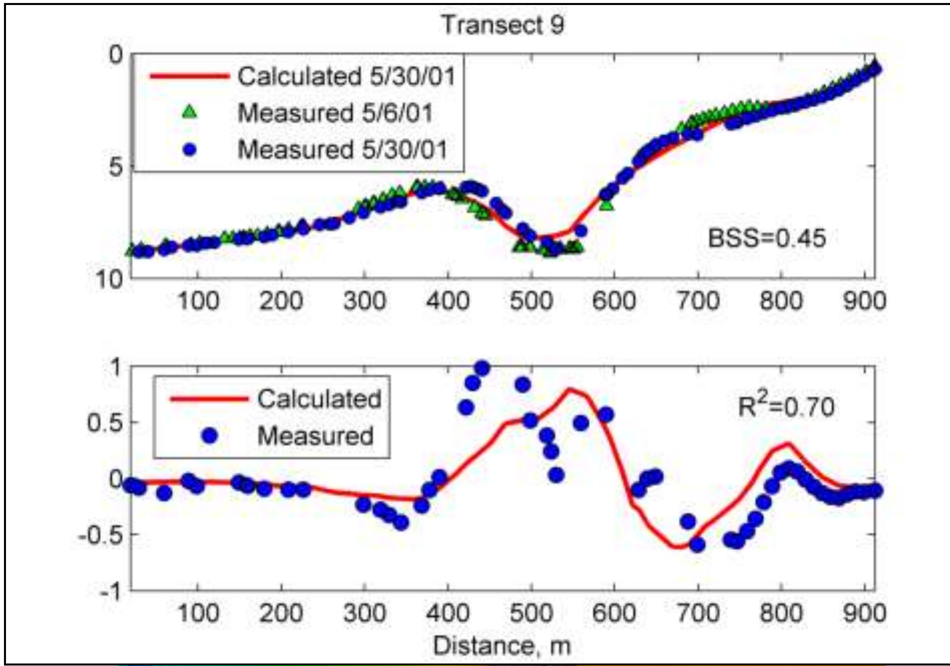
- Areas of calculated deposition and erosion are highlighted with black polygons
- Similar erosional and depositional trends
  - Erosion of outer bar
  - Deposition at inner bar face
  - Erosion of inner trough face
- Measured bed change shows more variability



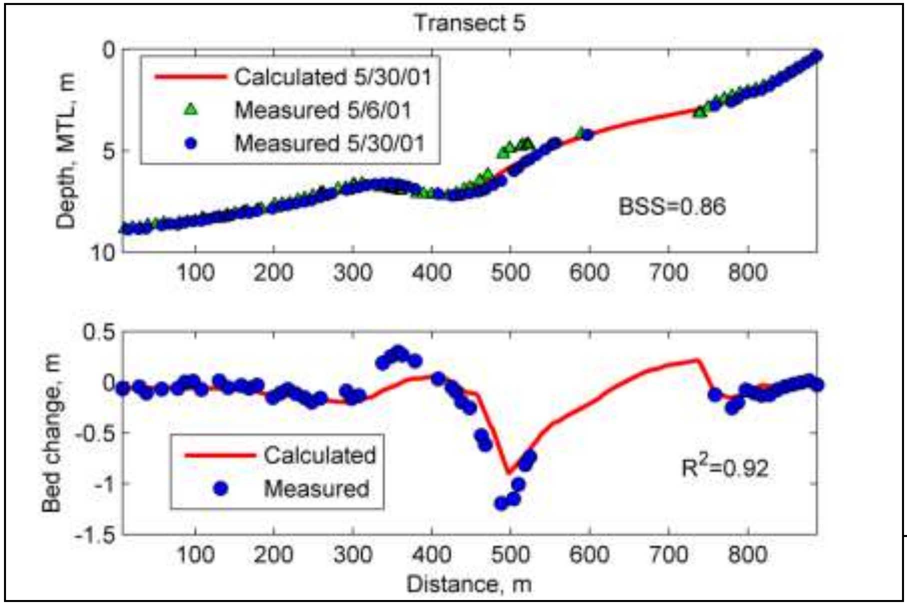
Transects



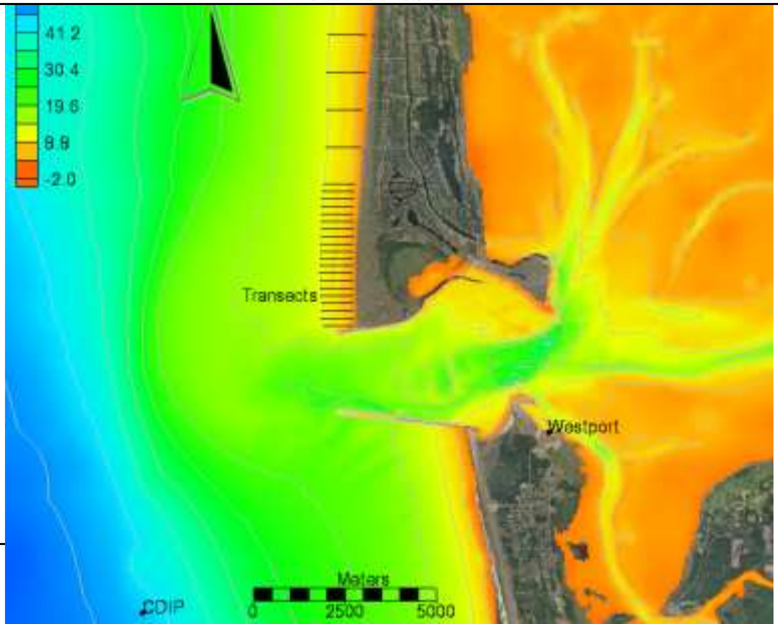
Transect 1
Calculated 5/30/01
Measured 5/6/01
Measured 5/30/01
BSS=0.40
Depth, MTL, m
Calculated
Measured
R^2=0.81
Bed change, m
Distance, m



Transect 9
Calculated 5/30/01
Measured 5/6/01
Measured 5/30/01
BSS=0.45
Calculated
Measured
R^2=0.70
Distance, m



Transect 5
Calculated 5/30/01
Measured 5/6/01
Measured 5/30/01
BSS=0.86
Depth, MTL, m
Calculated
Measured
R^2=0.92
Bed change, m
Distance, m



Transects
Westport
CDIP
Meters
0
2500
5000

CRESTS3D (Coastal, River, and Estuarine Simulation Tool System)

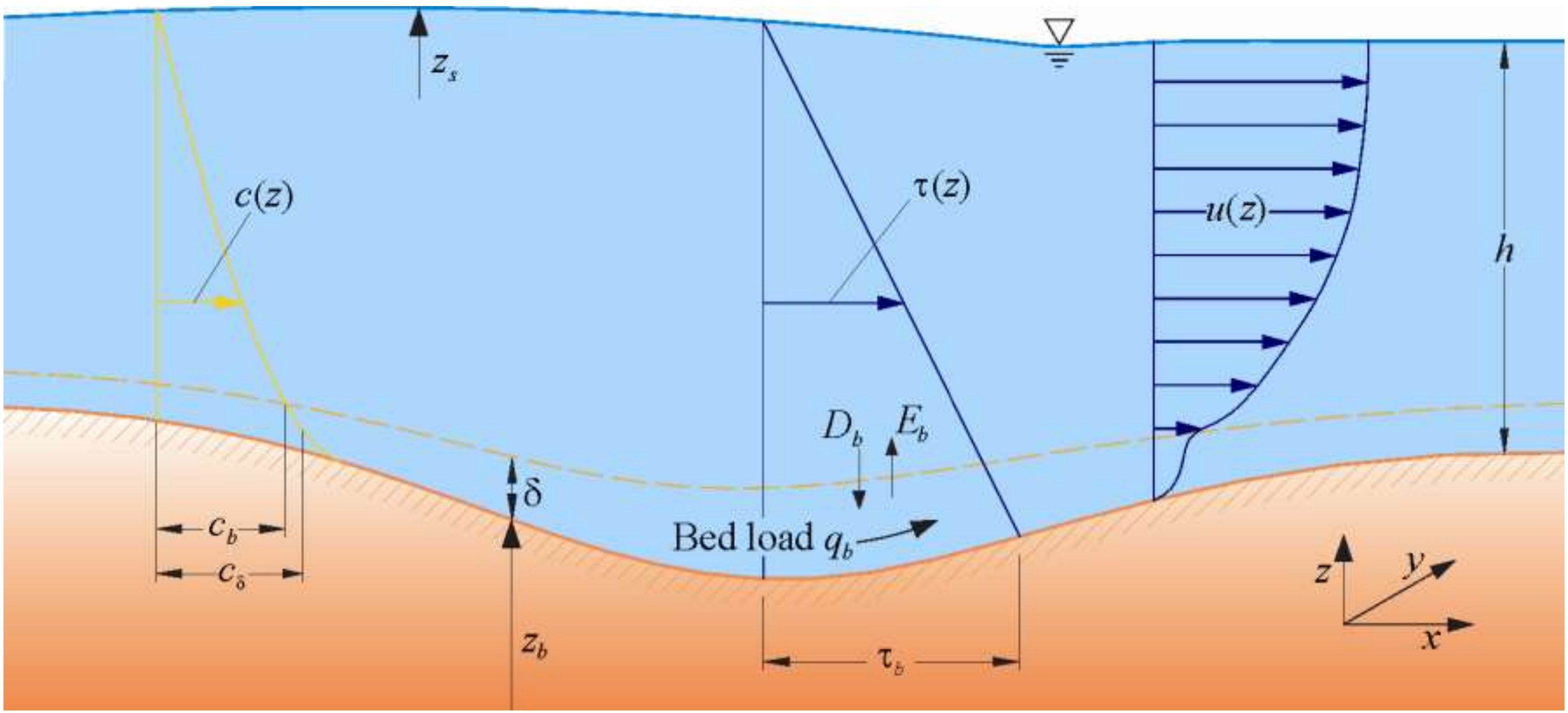
Weiming Wu

Clarkson University, NY, USA

3-D Model

- CRESTS
  - Coastal, Riverine and Estuarine Simulation Tool System
- A Phase-Averaged 3-D Shallow Water Flow Model
- Finite volume method
- Coupled with CMS-Wave
- Coded with CMS Flow Model
- SMS as GUI

Sketch of Flow and Sediment Transport



3-D Shallow Water Flow Equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\frac{\partial u}{\partial t}+\frac{\partial (uu)}{\partial x}+\frac{\partial (vu)}{\partial y}+\frac{\partial (wu)}{\partial z}=-\frac{1}{\rho}\frac{\partial p_a}{\partial x}-\frac{1}{\rho}\left(\rho_0 g\frac{\partial \eta}{\partial x}+g\int_z^{\eta}\frac{\partial \rho}{\partial x}dz\right)$$
$$+\frac{\partial}{\partial x}\left(\nu_{tH}\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu_{tH}\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\nu_{tV}\frac{\partial u}{\partial z}\right)-\frac{1}{\rho}\frac{\partial S_{xx}}{\partial x}-\frac{1}{\rho}\frac{\partial S_{xy}}{\partial y}-\frac{1}{\rho}f_x+f_c v$$

,

$$\frac{\partial v}{\partial t}+\frac{\partial (uv)}{\partial x}+\frac{\partial (vv)}{\partial y}+\frac{\partial (wv)}{\partial z}=-\frac{1}{\rho}\frac{\partial p_a}{\partial y}-\frac{1}{\rho}\left(\rho_0 g\frac{\partial \eta}{\partial y}+g\int_z^{\eta}\frac{\partial \rho}{\partial y}dz\right)$$
$$+\frac{\partial}{\partial x}\left(\nu_{tH}\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu_{tH}\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(\nu_{tV}\frac{\partial v}{\partial z}\right)-\frac{1}{\rho}\frac{\partial S_{yx}}{\partial x}-\frac{1}{\rho}\frac{\partial S_{yy}}{\partial y}-\frac{1}{\rho}f_y-f_c u$$

Boundary Conditions

Clarkson UNIVERSITY defy convention

Free surface kinematic condition

$$\frac{\partial \eta}{\partial t} + u_h \frac{\partial \eta}{\partial x} + v_h \frac{\partial \eta}{\partial y} = w_h$$

Surface shear stress due to wind

$$\tau_{si} = \rho_a C_D W W_i$$

,

where ρ_a is air density, C_D is the wind drag coefficient, and W is the wind velocity. The drag coefficient is calculated using the formula of Hsu (1988) and modified for high wind speeds based on field data by Powell et al. (2003).

Bed shear stress

$$\tau_{bx} = \rho c_f u_b \sqrt{u_b^2 + v_b^2 + 0.5 U_{wm}^2}, \quad \tau_{by} = \rho c_f v_b \sqrt{u_b^2 + v_b^2 + 0.5 U_{wm}^2}$$

where u_b and v_b are the x- and y-velocities near the bed; c_f is the bed friction coefficient; and U_{wm} is the maximum orbital bottom velocity of wave.

Coupling with CMS-Wave Model

Spectral wave-action balance equation (Mase et al. 2005):

$$\frac{\partial N}{\partial t}+\frac{\partial (c_x N)}{\partial x}+\frac{\partial (c_y N)}{\partial y}+\frac{\partial (c_\theta N)}{\partial \theta}=\frac{\kappa_w}{2\sigma}\left[\frac{\partial}{\partial y}\left(CC_g\cos^2\theta\frac{\partial N}{\partial y}\right)-\frac{1}{2}CC_g\cos^2\theta\frac{\partial^2 N}{\partial y^2}\right]-\varepsilon_b N-Q_v+Q$$

where $N = E(x,y,\sigma,\theta,t)/\sigma$; E is the spectral wave density representing the wave energy per unit water surface area per frequency interval; σ is the wave angular frequency (or intrinsic frequency); θ is the wave angle relative to the positive x-direction; C and C_g are wave celerity and group velocity, respectively; c_x, c_y, and c_θ are the characteristic velocities with respect to x, y and θ, respectively; κ_w is an empirical coefficient; ε_b is a parameter for wave breaking energy dissipation; Q_v represents the wave energy loss due to vegetation resistance; and Q includes source/sink terms of wave energy due to wind forcing, bottom friction loss, nonlinear wave-wave interaction, etc.

$$c_x = C_g\cos\theta + U \qquad c_y = C_g\sin\theta + V$$

$$c_\theta=\frac{\sigma}{\sinh 2kh}\left(\sin\theta\frac{\partial h}{\partial x}-\cos\theta\frac{\partial h}{\partial y}\right)+\cos\theta\sin\theta\frac{\partial U}{\partial x}-\cos^2\theta\frac{\partial U}{\partial y}+\sin^2\theta\frac{\partial V}{\partial x}-\sin\theta\cos\theta\frac{\partial V}{\partial y}$$

Wave Radiation Stress

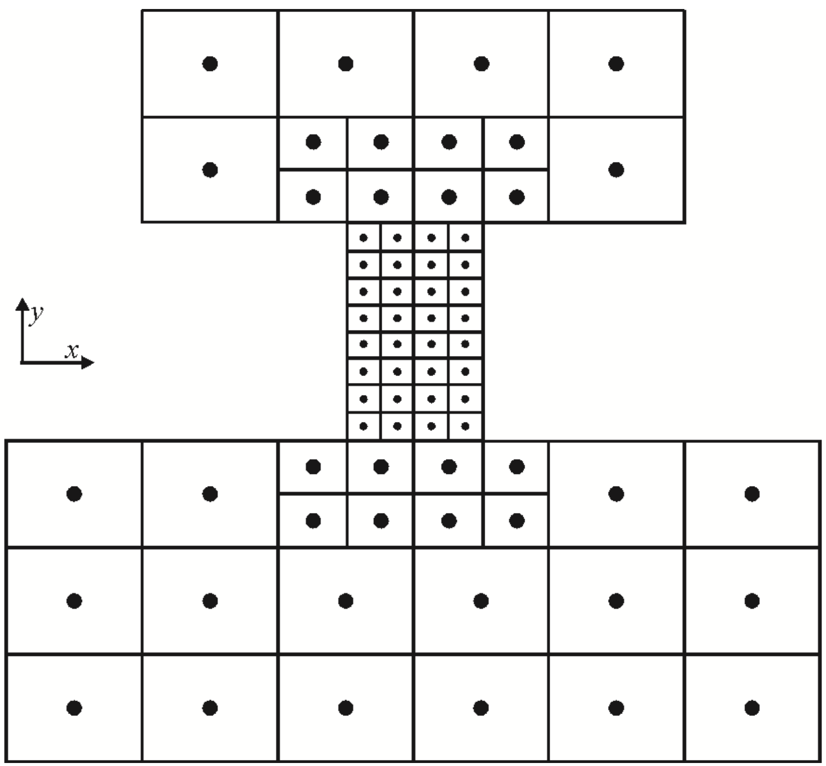
Formula of Mellor (2008)

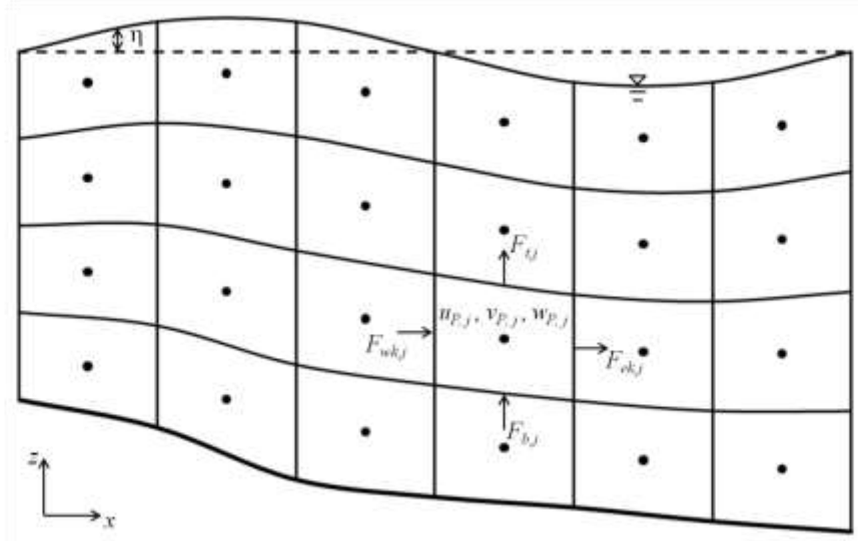
$$S_{ij}=\int_0^\infty\int_{-\pi}^{\pi}\left\{k(f)E(f,\theta)\left[\frac{k_i(f)k_j(f)}{k(f)^2}\frac{\cosh^2 k(h+z')}{\sinh kD\cosh kD}-\delta_{ij}\frac{\sinh^2 k(h+z')}{\sinh kD\cosh kD}\right]+\delta_{ij}E_D(f,\theta)\right\}d\theta df$$

where E is the wave energy, k is the wave number, θ is the angle of wave propagation to the onshore direction, f is the wave frequency, h is the still water depth, D is the total water depth, z' is the vertical coordinate referred to the still water level, and E_D is a modified Dirac delta function which is 0 if $z \neq \eta$ and has the following quantity:

$$\int_{-h}^{\eta+} E_D dz = E/2$$

3-D Mesh System





Quadtree rectangular in horizontal, and σ coordinate in vertical

Hybrid triangular and quadrilateral grid version in the horizontal is under development)

Numerical Solution Methods

- Finite volume method;
- Fully implicit;
- Non-staggered (collocated) grid;
- SIMPLEC, with under-relaxation;
- Rhie and Chow's (1983) momentum interpolation for interface fluxes;
- Upwind schemes:
  - Hybrid, Exponential, HLPA
- Solvers:
  - GMRES
- Drying and wetting: "Freezing" dry nodes.

3-D Sediment Transport Model

Suspended Load Transport

$$\frac{\partial c_k}{\partial t} + \frac{\partial\left[\left(u_j - \omega_{s,k}\delta_{j3}\right) c_k\right]}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_c}\frac{\partial c_k}{\partial x_j}\right)$$

(k=1, 2, …, N)

Bed Load Transport

$$\frac{\partial\left(q_{bk}/u_{bk}\right)}{\partial t} + \frac{\partial\left(\alpha_{bxk} q_{bk}\right)}{\partial x} + \frac{\partial\left(\alpha_{byk} q_{bk}\right)}{\partial y} + \frac{1}{L}\left(q_{bk} - q_{b*k}\right) = 0$$

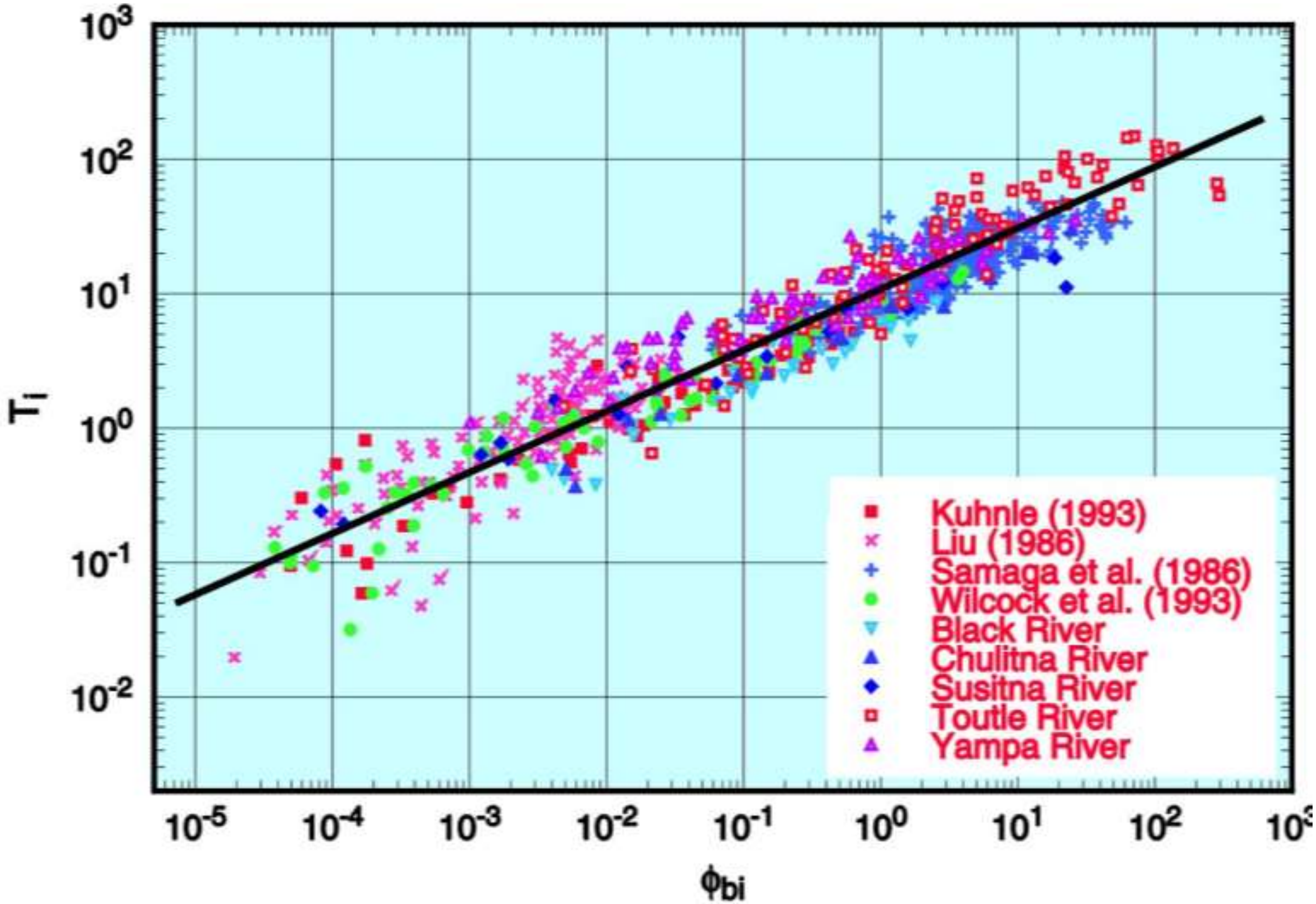
Bed Change

$$\left(1 - p'_m\right)\frac{\partial z_{bk}}{\partial t} = D_{bk} - E_{bk} + \frac{1}{L}\left(q_{bk} - q_{b*k}\right)$$

Bed Material Mixing

$$\frac{\partial(\delta_m p_{bk})}{\partial t} = \frac{\partial z_{bk}}{\partial t} + p^*_{bk}\left(\frac{\partial \delta_m}{\partial t} - \frac{\partial z_b}{\partial t}\right)$$

Wu et al. (2000) Bed Load Formula

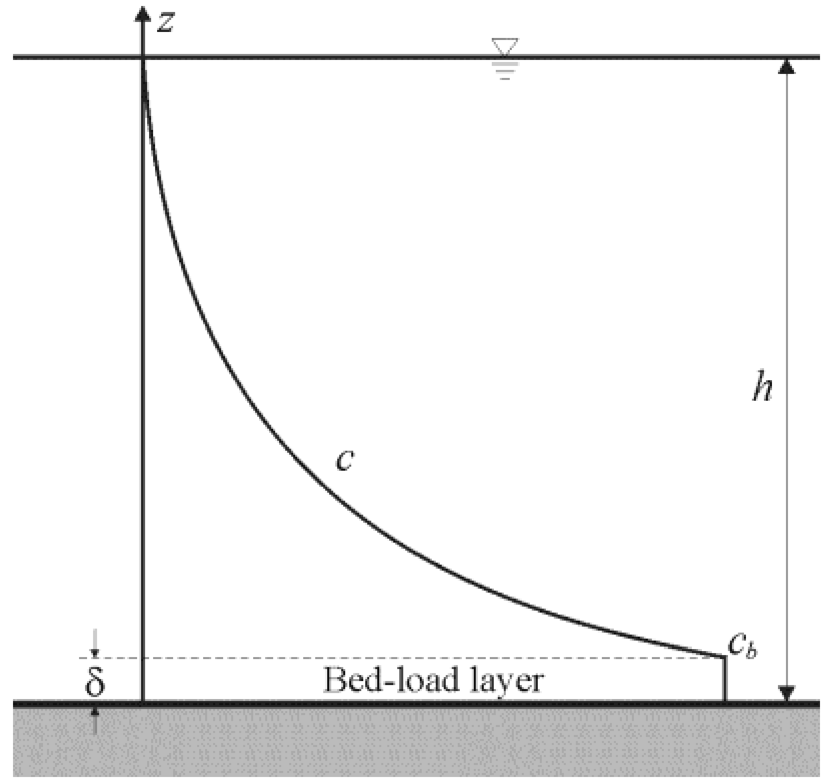


Extended to Coastal Sedimentation by Wu and Lin (2014, Coastal Engineering)

Near-Bed Suspended-load Concentration

Near-bed suspended-load concentration is related to bed-load transport rate:

$$c_{*bk} = \frac{q_{*bk}}{\delta u_{bk}}$$



Bed-load layer thickness:

$$\delta = \max\left(2.0d_{50}, 0.5\Delta_r, 0.01h\right)$$

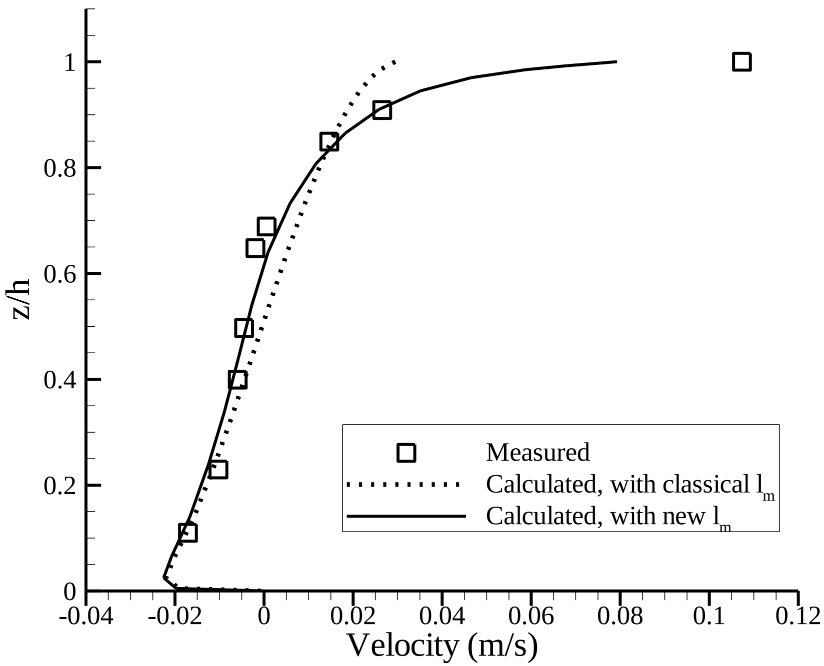
Bed-load velocity:

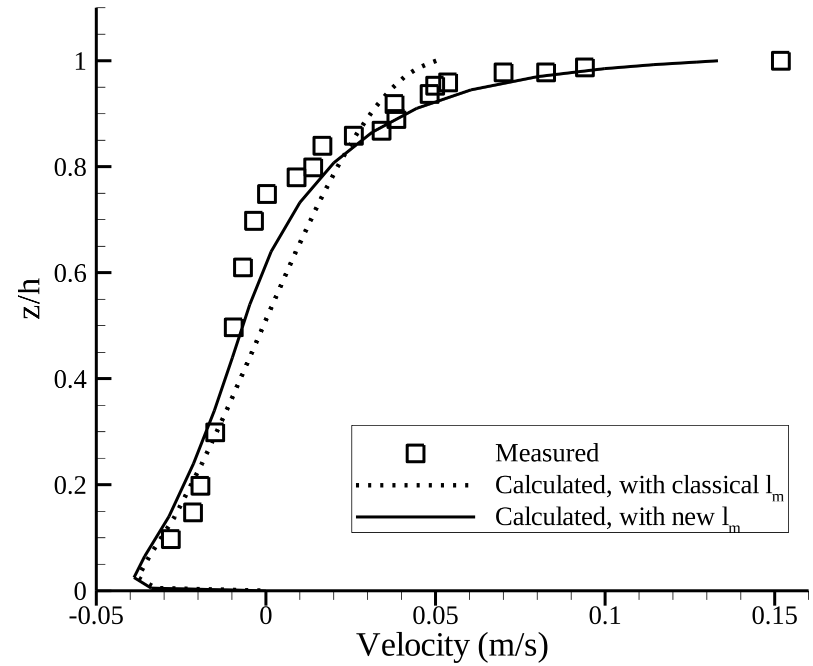
$$\frac{u_{bk}}{\sqrt{(\rho_s/\rho - 1)gd_k}} = 1.64\left(\frac{\tau_b'}{\tau_{cri,k}} - 1\right)^{0.5}$$

$$\Rightarrow \quad c_{*bk} = \frac{0.0032}{\delta} p_{bk} d_k \left(\frac{\tau'_b}{\tau_{cri,k}} - 1\right)^{1.7}$$

Wind-Induced Currents

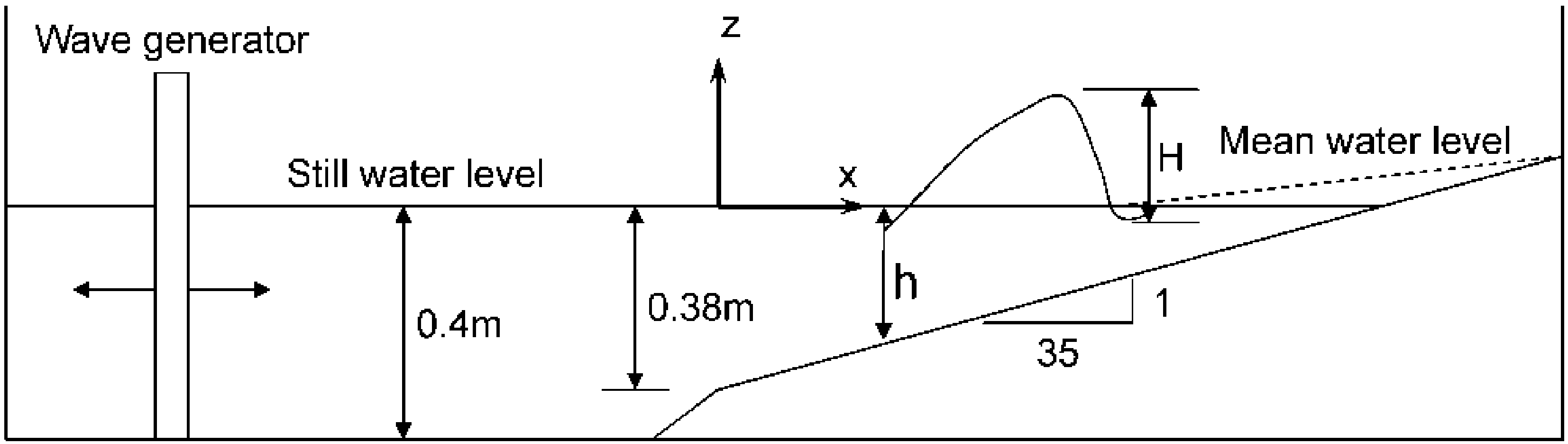
Experiments by Baines and Knapp (1965). Wind channel with a cross-section of 0.9144 m by 0.9144 m and a length of 12.8 m. The channel is discretized with square grid cells of side 0.061 m. Eighteen layers are used in the depth direction. The relative layer thickness (layer thickness over flow depth) from top to bottom is 0.005, 0.005, 0.01, 0.02, 0.03, 0.04, 0.05, 0.065, 0.085, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.05, 0.03 and 0.01. The bed friction coefficient c_f is 0.005.





Measured and simulated current velocities induced by wind with a speed of (left) 3.901 m/s and (right) 6.096 m/s

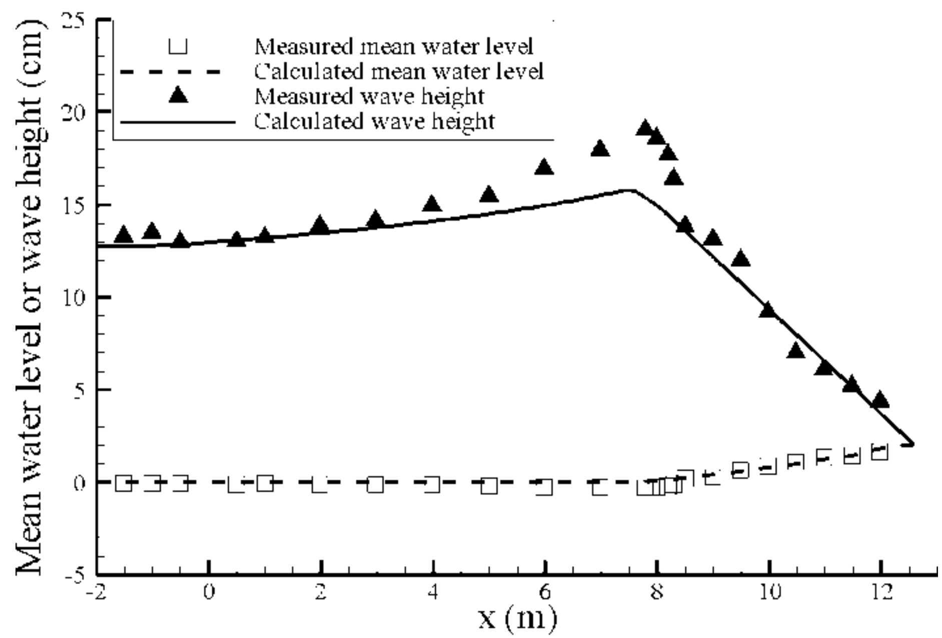
Cross-shore Undertow Current

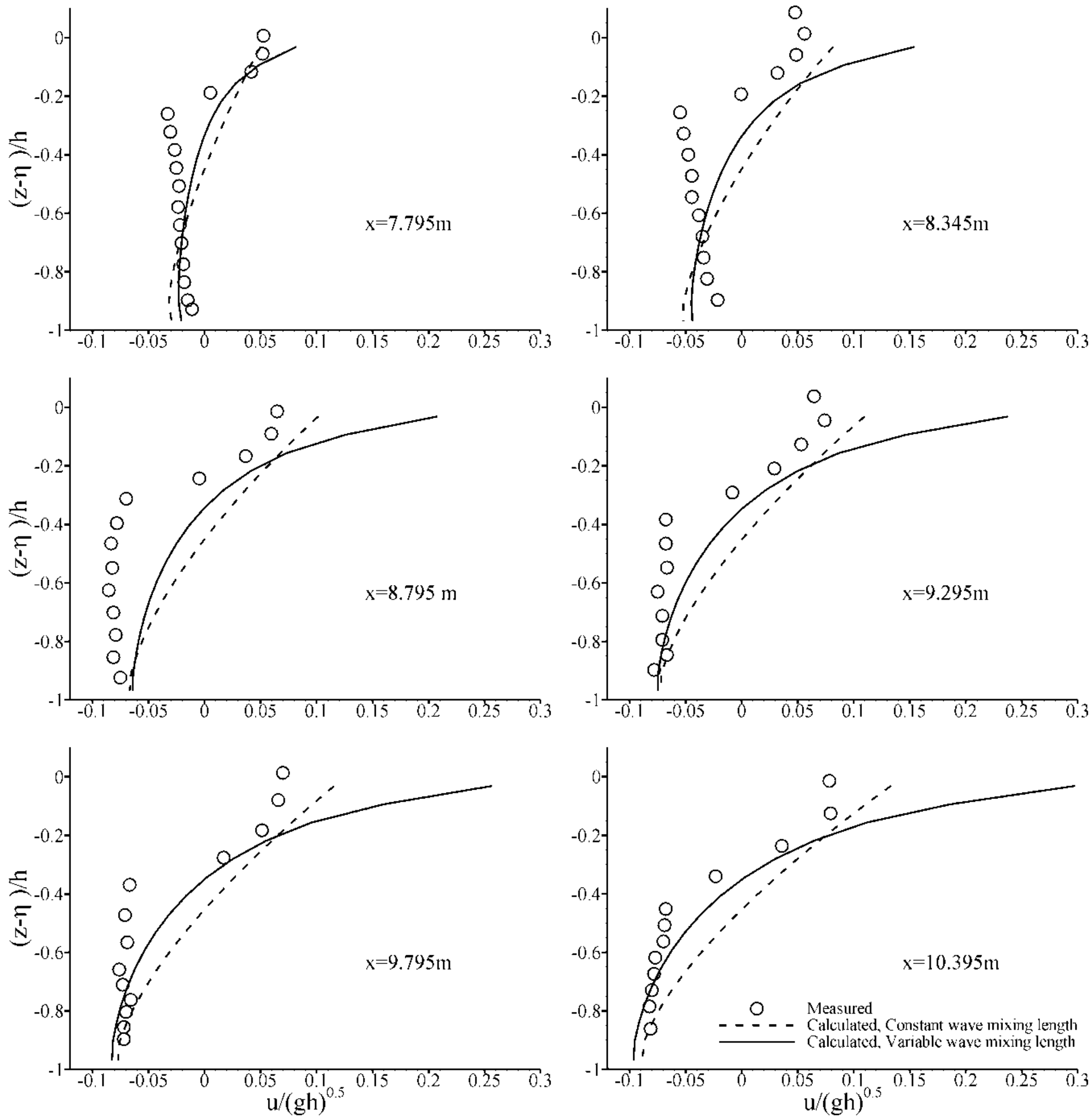


Experiment by Ting and Kirby (1994).

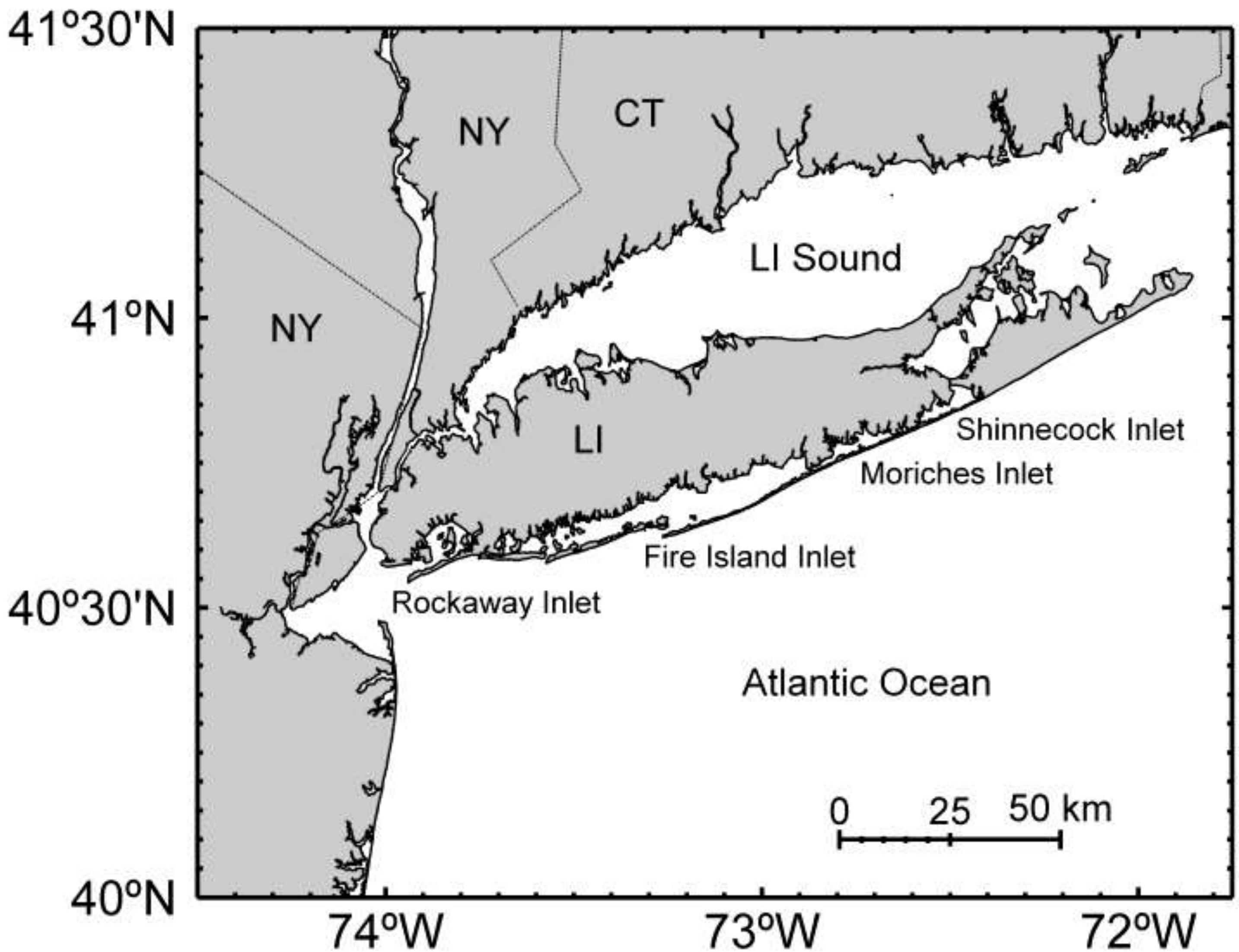
H=0.128 m, T=5 sec.

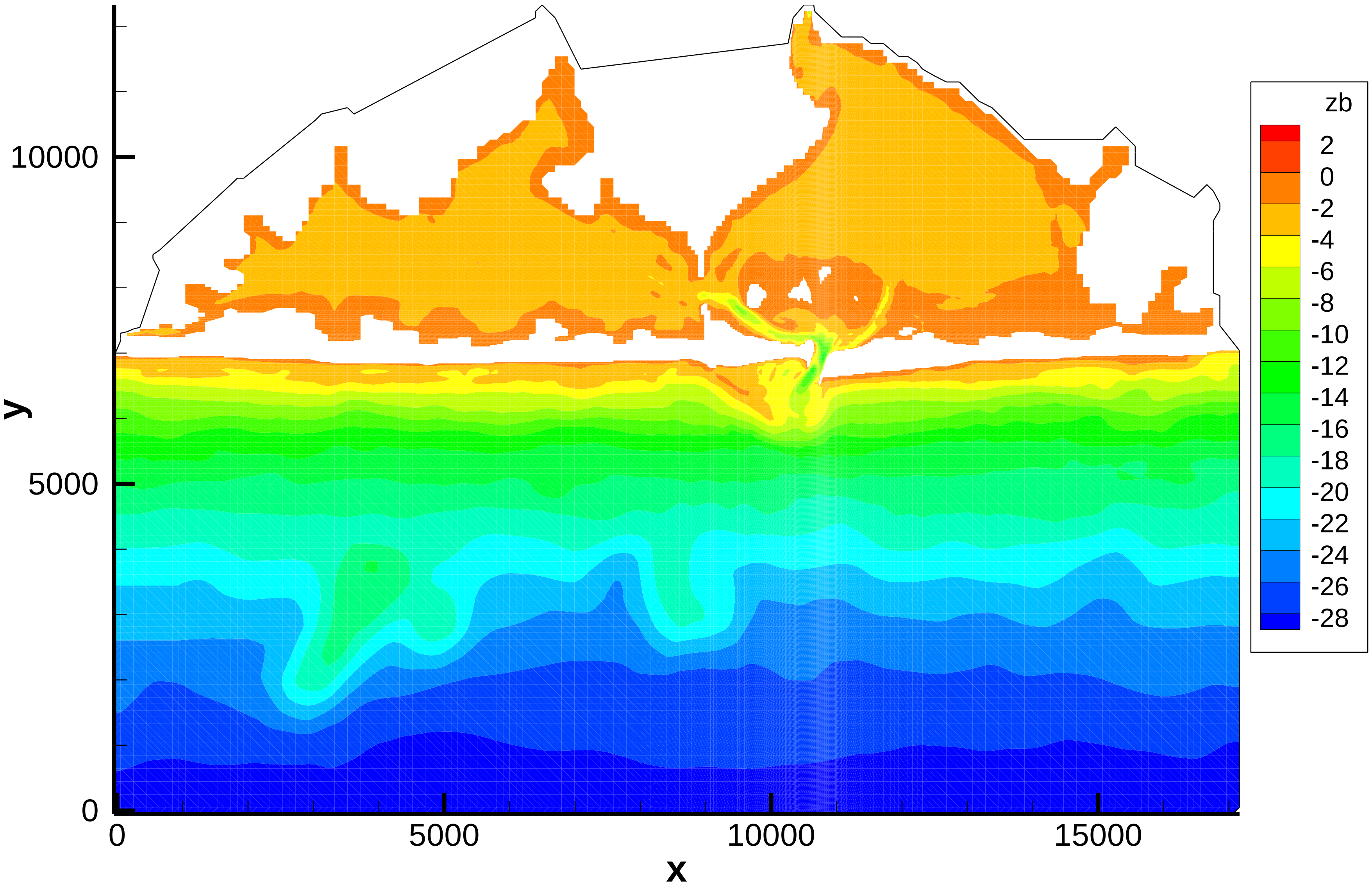
The cross-shore grid spacing is 0.5 m, and 16 layers with a uniform spacing are used in the vertical direction.



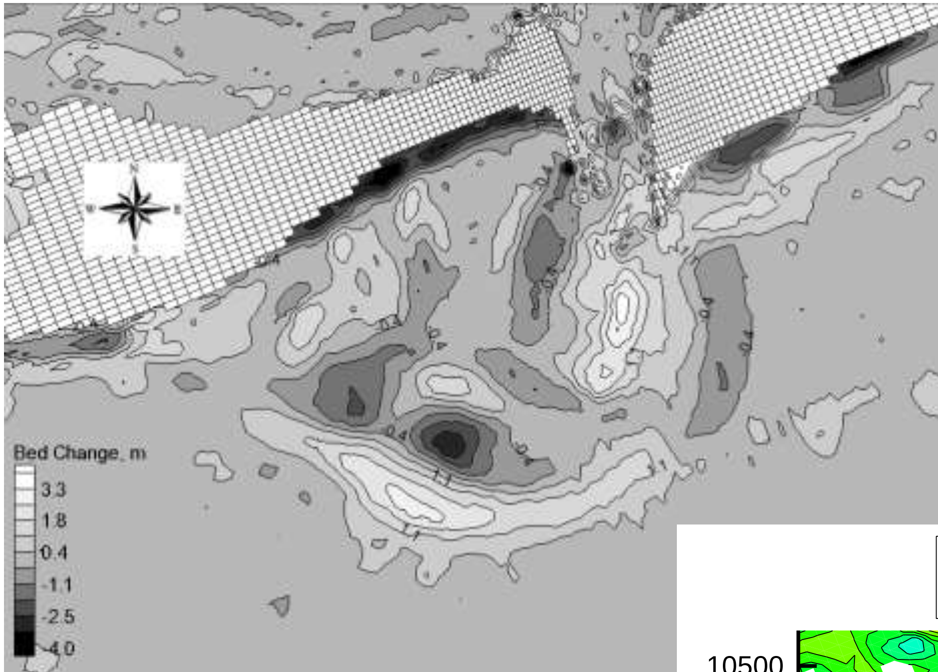

x=7.795m
x=8.345m
x=8.795 m
x=9.295m
x=9.795m
x=10.395m
(z-η)/h
u/(gh)^0.5
Measured
Calculated, Constant wave mixing length
Calculated, Variable wave mixing length

Shinnecock Inlet Case

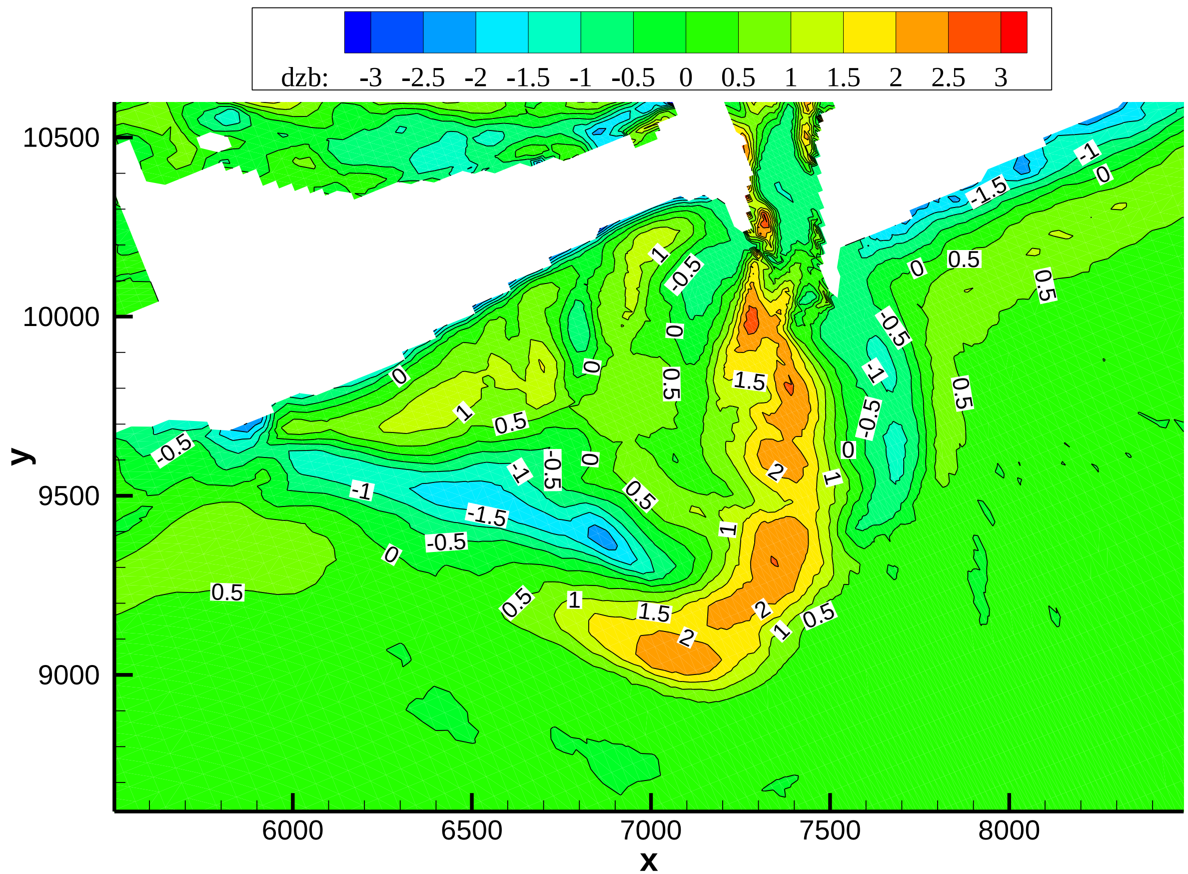




y
10000
5000
0
0
5000
10000
15000
x
zb
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
-24
-26
-28



Measured (left) and Calculated (right) Morphology Changes between August 1997 and May 1998



Summary

- A 3-D shallow water flow model has been developed for coastal sedimentation.
- A modified mixing length model is used for turbulence closure.
- The flow model is coupled with CMS-Wave model.
- The model equations are solved with a finite-volume method based on quadtree-rectangular mesh in the horizontal and σ coordinate in the vertical.
- The sediment transport model considers multiple-sized, total-load transport.
- The model has been tested using lab and field measurements.

Publications Related

W. Wu and S. S.Y. Wang (2004). "Depth-averaged 2-D calculation of tidal flow, salinity and cohesive sediment transport in estuaries," Int. J. Sediment Research, 19(3), 172–190.

A. Sanchez, W. Wu, H. Li, M. Brown, C. Reed, J.D. Rosati, and Z. Demirbilek (2014). "Coastal Modeling System: mathematical formulations and numerical methods." ERDC/CHL TR-14-2, Coastal and Hydraulics Laboratory, U.S. Army Engineer Research and Development Center, Vicksburg, MS.

W. Wu, A. Sanchez, and M. Zhang (2011). "An implicit 2-D shallow water flow model on unstructured quadtree rectangular mesh." Journal of Coastal Research, Special Issue, No. 59, pp. 15–26.

A. Sanchez (2013). "An implicit finite-volume depth-integrated model for coastal hydrodynamics and multiple-sized sediment transport." PhD Dissertation, the University of Mississippi, USA.

A. Sanchez and W. Wu (2011). "A non-equilibrium sediment transport model for coastal inlets and navigation channels." Journal of Coastal Research, Special Issue, No. 59, pp. 39–48.

W. Wu and Q. Lin (2011). "An implicit 3-D finite-volume coastal hydrodynamic model." Proc., 7th Int. Symposium on River, Coastal and Estuarine Morphodynamics, September 6-8, Beijing, China.